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Entropic Accelerating Universe

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To accommodate the observed accelerated expansion of the universe, one popular idea is to invoke a driving term in the Friedmann-Lemaître equation of dark energy which must then comprise 70% of the present cosmological energy density. We propose an alternative interpretation which takes into account the temperature intrinsic to the information holographically stored on the screen which is the surface of the universe. Dark energy is thereby obviated and the acceleration is due to an entropic force naturally arising from the information storage on a surface screen. We consider an additional quantitative approach based upon the entropy and surface terms usually neglected in general relativity and show that this leads to the entropic accelerating universe.

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I. INTRODUCTION

The most important observational advance in cosmology since the early studies of cosmic expansion in the 1920's was the dramatic and unexpected discovery, in the waning years of the twentieth century, that the expansion rate is accelerating. This was first announced in February 1998, based on the concordance of two groups' data on Supernovae Type 1A [1, 2]

A plethora of subsequent experiments concerning the Cosmic Microwave Background (CMB), Large Scale Structure (LSS), and other measurements have all confirmed the 1998 claim for cosmic acceleration. There have been many attempts to avoid the conclusion of the cosmic acceleration. Typically they involve an ingenious ruse which assigns a special place to the Earth in the Universe, in a frankly Ptolemaic manner and in contradiction to the well-tested and time-honored cosmological principle at large distance. We find these to be highly contrived and *ad hoc*.

We therefore adopt the position that the accelerated expansion rate is an observed fact which we, as theorists, are behooved to interpret theoretically with the most minimal set of additional assumptions.

II. INTERPRETATION AS DARK ENERGY

On the basis of general relativity theory, together with the cosmological principle of homogeneity and isotropy, the scale factor $a(t)$ in the FRW metric satisfies [3, 4] the Friedmann-Lemaître equation

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho \tag{1}$$

where we shall normalize $a(t_0) = 1$ at the present, time $t = t_0$, and ρ is an energy density source which drives the expansion of the universe. Two established contributions to ρ are ρ_m from matter (including dark matter) and ρ_γ radiation, so that

$$\rho \supseteq \rho_m + \rho_\gamma \tag{2}$$

with $\rho_m(t) = \rho_m(t_0)a(t)^{-3}$ and $\rho_\gamma(t) = \rho_\gamma(t_0)a(t)^{-4}$.

For the observed accelerated expansion, the most popular approach is to add to the sources, in Eq.(1), a dark energy term $\rho_{DE}(t)$ with

$$\rho_{DE}(t) = \rho_{DE}(t_0)a(t)^{-3(1+\omega)} \tag{3}$$

where $\omega = p/\rho c^2$ is the equation of state. For the case $\omega = -1$, as for a cosmological constant, Λ, and discarding the matter and radiation terms which are smaller we can easily integrate the Friedmann-Lemaître equation to find

$$a(t) = a(t_0)\ e^{Ht} \tag{4}$$

where $\sqrt{3}H = \sqrt{\Lambda} = \sqrt{8\pi G\rho_{DE}}$.

By differentiation of Eq. (4) with respect to time p times we obtain for the p^{th} derivative

$$\frac{d^p}{dt^p}a(t)|_{t=0} = (H)^p \tag{5}$$

Therefore, if $\Lambda > 0$ is positive, as in a De Sitter geometry, not only is the acceleration ($p = 2$) positive and non-zero, but so are the jerk ($p = 3$), the snap ($p = 4$), the crackle ($p = 5$), the pop ($p = 6$) and all $p \geq 7$.

The insertion of the dark energy term Eq. (3) in Eq. (1) works very well as a part of the ΛCDM model. However, it is an *ad hoc* procedure which gives no insight into what dark energy is.

With this background, we shall now move to a different explanation for the accelerated expansion which obviates any dark energy, including scalar fields or a cosmological constant.

III. INTERPRETATION AS ENTROPIC FORCE

We now adopt a different approach, with no dark energy, where instead the central role is played by the ideas of information and holography, entropy and temperature.

The first and only assumption is holography, by which we understand that all the information about the universe is encoded on a screen, here taken as the two-dimensional surface of the universe.

At this horizon, there is a horizon temperature, T_β, which we can estimate as

$$T_\beta = \frac{\hbar}{k_B}\frac{H}{2\pi} \sim 3 \times 10^{-30} K. \tag{6}$$

Such a temperature is closely related to the de Sitter temperature[1]. More relevant to the central question is the fact that the temperature of the horizon screen leads to the concomitant entopic force and resultant acceleration $a_{Horizon}$ of the horizon given by the Unruh [5] relationship

$$a_{Horizon} = \left(\frac{2\pi c k_B T_\beta}{\hbar}\right) = cH \sim 10^{-9}\, m/s^2\,. \tag{7}$$

When T_β is used in Eq. (7), we arrive at a cosmic acceleration essentially in agreement with the observation.

From this viewpoint, the dark energy is non-existent. Instead there is an entropic force [6, 7] acting at the horizon and pulling outward towards the horizon to create the appearance of a dark energy component.

We shall next amplify on the distinction, and study more the entropy and surface screen considerations, showing that even the present fraction of the critical energy associated with acceleration can thereby be understood.

[1] We suspect, without rigor, that in the third law of thermodynamics the notion of absolute zero, $T = 0$, must be replaced by $T \geq T_\beta$, although this is not our present concern.

IV. ACCELERATION FROM THE ENTROPY AND SURFACE TERMS

We consider the least additional assumption is that general relativity is correct, and that it can be easily understood and derived from a variational principle using the action. The ingredient that is usually neglected is the surface term. We show that, under reasonable assumptions, this surface term leads to an acceleration term in the Friedmann-Lemaître equations. There is a solution to the acceleration equation that evolves from a decelerating to an accelerating phase.

The Einstein-Hilbert action including the surface term and a matter action is (schematically)

$$I = \int_M (R + \mathcal{L}_m) + \frac{1}{8\pi} \oint_{\partial M} K \tag{8}$$

where R is the scalar curvature, $\mathcal{L}_m$ is the matter and field Lagrangian, and K is the trace of the extrinsic curvature of the boundary [8]. The application of variational procedures then produces the usual Einstein equations for general relativity with the addition of a surface energy term:

$$\text{Curvature of Space} - \text{Time} \quad \textit{proportional} \quad \text{to the Stress} - \text{Energy Content} \ + \ \text{Surface Terms}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \qquad = \qquad \frac{8\pi G}{c^4} T_{\mu\nu} + \ Surface\ terms \tag{9}$$

Typically the surface terms are neglected though they have not been shown to be negligible. This would in the case of spherical symmetry and homogeneity lead to the Friedmann-Lemaître equations:

$$\text{Scale factor acceleration} \ = \ \text{Energy Content deceleration} \ + \text{acceleration from Surface Terms}$$

$$\frac{\ddot{a}}{a} \ = \ -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2} \right) + a_{Surface}/d_H \tag{10}$$

Now, there are a number of approaches to determine what are the surface terms. Here we consider a simple possibility. We would anticipate that the integral of the trace of the intrinsic curvature would be of order $6(2H^2 + \dot{H})$ so that the term would be approximately $\frac{6(2H^2+\dot{H})}{8\pi} \sim \frac{3}{2\pi}(H^2 + \dot{H}/2)$. We can also take an approach motivated by entropic ideas and see that these lead to a natural out come of this and thus of a slowly expanding accelerating universe.

For an event horizon, it is well known that there is an associated curvature and temperature, and that these two quantities are related. The temperature T is given by the Unruh, de Sitter, or Hawking temperature prescriptions (except for a pesky factor of two difference between the Hawking temperature and the other two due to location of evaluation of the temperature - at the horizon or remote). We can then associate the surface entropy or surface term with its temperature and its acceleration. Using the relations we find, see Eq. (7) for the horizon acceleration

$$a_{surface} = a_{entropic} = cH \tag{11}$$

where $H = \dot{a}/a$ is the Hubble expansion rate. If we have a scale, which is naturally and necessarily the Hubble horizon scale, $d_H = c/H$, for our cosmological treatment, then we can complete the Friedmann-Lemaître acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + H^2 \,. \tag{12}$$

This is remarkably like the surface term order of magnitude estimate except for the $3/2\pi$ factor. With the Hawking temperature description the coefficient would have been 1/2. There is some freedom here and we chose the value that leads to nice equation in the two limiting cases. It is easy to show that if the H^2 is highly dominant over the $\frac{4\pi G}{3}(\rho + 3P/c^2)$, the solution to the equation is simply de Sitter $a(t) = a(t_o)e^{H(t-t_0)}$. The alternate equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{3}{2\pi}H^2 + \frac{3}{4\pi}\dot{H} \tag{13}$$

may work better for fitting the data (see §V) or a rigorous derivation but does not have the simplicity of Eq. (12).

A. Entropic Force Considerations

We bolster this and particularly the argument of the previous section by showing the entropic force and thus entropic acceleration is directed at the entropic information encoded on a screen. The next portion derives an expression for the pressure, which is negative and thus a tension in the direction of the screen. The following results rely on simple principles of holography and thermodynamic and are not dependent on the specific model inspired by the surface terms in Eq. (8).

The entropy on the screen, e.g. the horizon, is

$$S_H = \frac{k_B c^3}{G\hbar}\frac{A}{4} = \frac{k_B c^3}{G\hbar}\pi R_H^2 = \frac{k_B c^3}{G\hbar}\pi\left(\frac{c}{H}\right)^2 \sim (2.6 \pm 0.3) \times 10^{122} k_B \tag{14}$$

Increasing the radius R_H, by Δr, increases the entropy by ΔS_H according to

$$\Delta S_H = \frac{k_B c^3}{G\hbar} 2\pi R_H \Delta r = \frac{k_B c^3}{G\hbar} 2\pi\left(\frac{c}{H}\right)\Delta r \sim (2.6 \pm 0.3) \times 10^{122} k_B \Delta r / R_H \tag{15}$$

The entropic force is simply

$$F_r = -\frac{dE}{dr} = -T\frac{dS}{dr} = -T_\beta \frac{dS_H}{dr} = -\frac{\hbar}{k_B}\frac{H}{2\pi}\frac{k_B c^3}{G\hbar}2\pi\left(\frac{c}{H}\right) = -\frac{c^4}{G} \tag{16}$$

where the minus sign indicates pointing in the direction of increasing entropy or the screen, which in this case is the horizon.

The pressure from entropic force exerted is

$$P = \frac{F_r}{A} = -\frac{1}{A}T\frac{dS}{dr} = -\frac{1}{A}\frac{c^4}{G} = -\frac{1}{4\pi c^2/H^2}\frac{c^4}{G} = -\frac{c^2 H^2}{4\pi G} = -\frac{2}{3}\rho_{critical}c^2 \tag{17}$$

This is, of course, close to the value of the currently measured dark energy / cosmological constant negative pressure (equals tension). In this case the tension does not arrive from the negative pressure of dark energy but from the entropic tension (outward pointing pressure) due to the (information) entropy content of the surface. This is equivalent to the outward acceleration $a_H = cH$ of Eq. (7).

If we want to use the form of the surface term, then we would find the equivalent $3P$ term would be given by $3P = -\frac{3}{\pi}\rho_{critical}c^2 \times (1 + \frac{1}{4}\frac{\dot{H}}{H^2})$ versus the $3P = -2\rho_{critical}c^2$ from above.

If we chose to put the information screens at smaller radii, then we would have found a proportionally smaller pressure, and an acceleration that decreases linearly with the radius, in accordance with our expected Hubble law. Thus, the acceleration of the universe simply arises as a natural consequence of the entropy of the universe, via the holographic principle.

V. DISCUSSION

We concluded with a demonstration that the entropic acceleration mechanism can provide a surprisingly remarkable fit the supernova data, assuming the simple form for the acceleration equation (13). Because we are using a metric theory of gravity, we may use the standard formula for the luminosity distance:

$$d_L(z; H(z), H_0) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{H(z')}\,, \tag{18}$$

where z is the redshift defined by $z + 1 \equiv a_0/a$.

For ΛCDM, the luminosity distance (18) can be written [1]:

$$d_L(z; \Omega_M, \Omega_\Lambda, H_0) = \frac{c(1+z)}{H_0\sqrt{|\kappa|}}\mathcal{S}\left(\sqrt{|\kappa|}\int_0^z dz' \left[(1+z')^2(1+\Omega_M z') - z'(2+z')\Omega_\Lambda\right]^{-\frac{1}{2}}\right) \tag{19}$$

where, $\mathcal{S}(x) \equiv \sin(x)$ and $\kappa = 1 - \Omega_{tot}$ for $\Omega_{tot} > 1$ while $\mathcal{S}(x) \equiv \sinh(x)$ with $\kappa = 1 - \Omega_{tot}$ for $\Omega_{tot} < 1$ while $\mathcal{S}(x) \equiv x$ and $\kappa = 1$, for $\Omega_{tot} = 1$. For the ΛCDM models we take $\Omega_{tot} = \Omega_M + \Omega_\Lambda$. Here we have defined $\Omega_M \equiv \rho_M/\rho_c = 8\pi G\rho_M/3H^2$ and $\Omega_\Lambda = \Lambda/3H^2$ where ρ_c is the critical energy density. The results are that the entropic acceleration models we consider can provide excellent fits to the data as can be seen from Fig. (1). [2] We suspect a complete model may be further constrained by consideration of Big-Bang Nucleosynthesis, also possibly by precision data concerning the equivalence principle.

We have discussed a theory underlying the accelerated expansion of the universe based on entropy and entropic force. This approach, while admittedly heuristic, provides a physical understanding of the acceleration phenomenon which was lacking in the description as dark energy.

The entropy of the universe has received some recent attention [9, 10], in part because it relates to the feasibility of constructing a consistent cyclic model. For example, the cyclic model in [11], assuming its internal consistency will indeed be fully confirmed, provides

[2] In solving the equations for the fitting we have assumed the standard scaling behavior for matter and radiation. This scaling behavior is now determined only in part by the Friedmann-Lemaître equation and we do not propose any novel form for this constraint yet; rather, we assume that the standard scaling applies and should provide an adequate approximation for the purposes of Fig. (1).

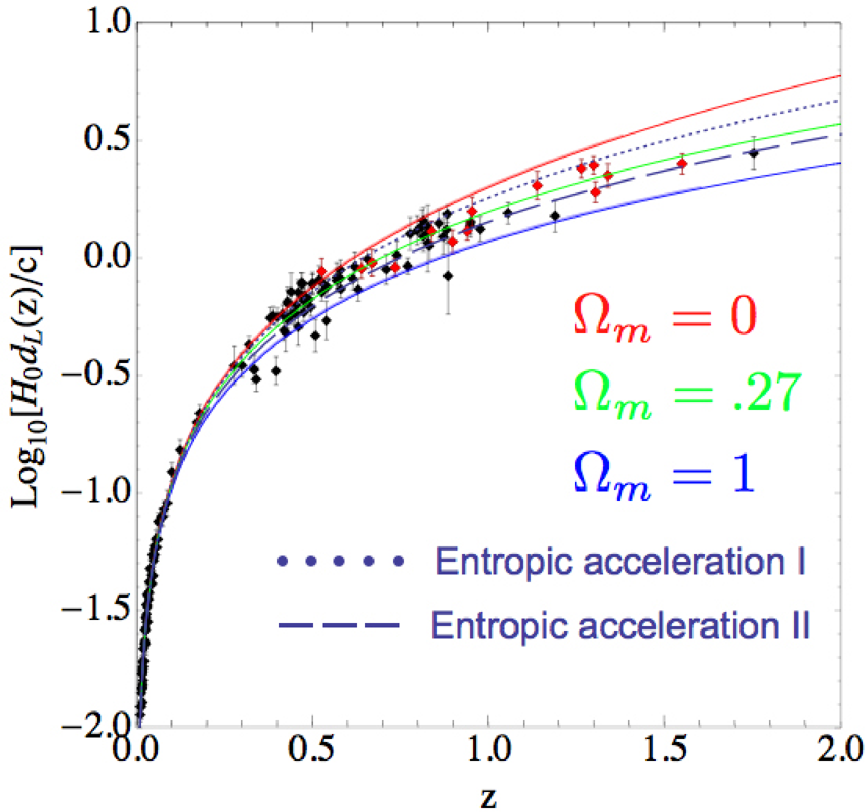

FIG. 1: Comparison of entropic acceleration and several ΛCDM models. The supernovae data points are plotted with error bars and the data is taken from [13]. The luminosity distance d_L for the entropic models I (Eq. (12)) and II (Eq. (13)) are denoted by the dotted and dashed (blue) curves respectively. The theoretical predictions for ΛCDM are represented by the solid curves.

the solution to a difficult entropy question originally posed, seventy-five years earlier, by Tolman [12].

The accelerated expansion rate is no longer surprising. It is the inevitable consequence of the holographic information storage on the surface screen of the universe. An interesting question [14] is: how does this entropic viewpoint of cosmic acceleration impact on inflationary theory?

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