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ABSTRACT

This paper shows by way of an example that the worst case condition that has been used over the years for intersystem interference calculations leads to considerably pessimistic results. The intersystem interference is considered as a statistical phenomenon and as such it is treated within the general context of random processes. The particular example that is studied in some detail for illustrating the methodology is a set of Single Channel Per Carrier (SCPC) systems interfering into a wideband channel which for simplicity is chosen to be an FDM/FM system. The interference is considered to be the result of the random distribution of SCPC carriers of the various systems over the bandwidth of the interfered-with network and of the statistical nature of sidelobe gain of the earth station antennas. This shows that for a typical set of interference conditions the interference power is less, by 6 dB with a confidence of 99%, than the worst case power obtained by a deterministic approach. This result allows for a more efficient orbit/spectrum utilization for this type of satellite systems. It is pointed out that similar methods can be used to analyze intersystem interference when other statistical link parameters are involved.

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subject: A Statistical Approach to the Interference Reduction of a Class of Satellite Transmission Systems

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from: S. C. Moorthy
P. Stavroulakis

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I. INTRODUCTION

A fundamental limitation in the utilization of the geostationary orbit is the mutual interference generated by satellite networks. The rapid expansion of geostationary satellite systems over the last 10 years has accentuated the problem and the need for the establishment of equitable regulations for sharing the geostationary orbit is greater than ever. Some of the areas that are being studied as potential contributors to the reduction of the intersystem interference are: 1) better satellite station-keeping, 2) limitations on satellite and earth station sidelobe patterns, 3) carrier energy dispersal, 4) adoption of an angle dependent interference criterion to name a few.

Intersystem interference, however, is a statistical phenomenon and as such it must be treated within the general context of random processes.

The objective of this paper is to demonstrate by analysis that benefits can be derived from the statistical nature of interference for a class of satellite transmission system. The particular system to be studied comprises a number of Single Channel Per Carrier (SCPC) systems interfering into a wideband channel which for simplicity is chosen to be a FDM/FM system. The analysis considers the interference of the multiple SCPC system interferer into the FDM/FM system to be the result of the random distribution of the SCPC carriers of the various systems over the bandwidth of the interfered-with network and of the statistical nature of sidelobe gain of the earth station antennas. Since there is no reason to believe that any interdependence between these two exists, we shall assume that each one can be analyzed separately as far as its statistical interference characteristics are concerned. Numerical results are obtained by computer simulation.

II. PROBLEM DEFINITION

SCPC systems are characterized by transmission of a large number (several hundred to a thousand) of frequencies within a satellite transponder. In computing the interference of SCPC systems into an FDM system on adjacent satellites the SCPC carriers that contribute the most are those that are closest to the location of the desired baseband channel in the RF spectrum of the wanted FDM system. The SCPC systems under consideration are assumed to use voice activated transmission (VOX) and demand assigned multiple access (DAMA) form of carrier allocation. A characteristic of this form of allocation is that only a small fraction (0.25 to 0.5) of the total number of channel slots are active at any given time. The objective of what follows is to determine how often a certain level of interference occurs considering various strategies of carrier assignment.

In processing a call request, the DAMA central control computer selects two frequencies from the total number of frequency (channel) slots N. There are several different schemes of making this selection: At the heart of all these schemes is a random selection of a number from 1 to N such that all numbers have the same probability of 1/N to be selected. Let us label this step 1. This is accomplished by the computer by using the random function generating routine, that most computers have in their library. Usually a call to this routine returns a number x uniformly distributed in the interval (0,1). A linear transformation of the type $y = (N-1)x+1$ provides a uniform distribution in the interval [1,N].

The selection of the two frequencies can be classified in two categories as ((a),(b)) described below.

(a) Paired Frequencies

The DAMA controller performs step 1, and obtains a slot number I. It checks to see whether I is already assigned. If it is already assigned it repeats step 1 until an unassigned I is obtained. It selects the frequency corresponding to I as one of the frequencies. The other frequency is obtained by adding a fixed offset of $\pm\beta\Delta F$ to the first frequency where ΔF is the spacing of the SCPC carrier slots and β is any integer from 1 to [N/2].

(b) Unpaired Frequencies

The first frequency is selected as above. In addition the second frequency is also selected in the same manner. Pairing of frequencies results in some hardware savings (e.g., elimination of a frequency synthesizer) at the earth stations. In addition sufficient randomness is retained so that intermodulation advantage for case (a) is the same as that for case (b). There are variations possible in the configuration of the N channel slots from which the DAMA controller selects frequencies. They can be classified as follows ((c), (d), (e)).

(c) Uniform Spacing

In this scheme N slots are spaced ΔF apart across the transponder.

(d) Staggered Spacing

This configuration is obtained by starting with uniform spacing (case (c)) and shifting the carriers on one half of the transponder by $\Delta F/2$. This causes the distribution of intermodulation noise to spread out.

(e) <u>Half Spacing</u>

Here there are (2N-1) channel slots. This is obtained by choosing the frequency synthesizer step to be $\Delta F/2$. Obviously two frequencies $\Delta F/2$ apart should not be selected by the DAMA controller; they have to be at least ΔF apart. However this scheme permits carriers to be separated by $3\Delta F/2$, $5\Delta F/2$... etc. which is not possible in (c). Step 1 discussed earlier should be modified to keep the carrier spacing no lower than ΔF. This scheme also causes the distribution of intermodulation noise to spread out.

It is seen that the combination of (a), (b), with (c), (d), and (e) lead to a number of strategies for carrier assignment by the DAMA controller. They all result in some sort of random distribution of carriers in the allowed channel slots. All these schemes are easy to implement on a computer program for simulation purposes and of course in the operating system. To compute the improvement in interference due to a random assignment strategy alone, we first define a reference interference case for which the carriers are present in all channel slots simultanecusly using the procedure that is outlined in the section that follows. Next the random experiment is performed in the simulation, as many times as it is statistically necessary to obtain good estimate of the expected value and the variance of the interference power.

As we mentioned before the interference power depends also on the antenna sidelobe gain pattern. It has been suggested in a C.C.I.R Study Group Document [1] that a statistical description of the antenna sidelobes based experimental data is possible. A Rayleigh distribution law has been used as a model. A recent paper [2] discussed the evaluation of interference levels based on the above representation of sidelobe gain and computed the exact distribution of the sum of two interferences.

This paper presents a method of calculating the interference power into a wideband channel (e.g., FDM/FM channel*) from a multiple SCPC satellite network based on the Reyleigh distribution law for the sidelobe gain. In particular, the interfering model of the Figure 1 will be assumed. The gain of the antenna at any angle θ from the main axis is then given by

$$f(g) = \frac{g}{\sigma_\theta^2} \exp\left(- \frac{g^2}{2\sigma_\theta^2}\right) , \quad g \geq 0 \tag{1}$$

In equation (1) σ_θ^2 is chosen such that for any angle θ the probability that the actual gain, g, exceeds the C.C.I.R. standard envelope, 32-25 logθ is 5%.** This gives a physical interpretation of the variable σ_θ. Hence,

* The analysis that follows considers only the downlink interference for simplicity in the illustration of the methodology but it applies equally to the uplink ingerference.

** This is in line with the assumptions used to derive this envelope

$$\text{Prob}(g \geq 32-25 \log\theta) = 0.05 \quad \text{or}$$

$$\int_{10^{3.2}\theta^{-2.5}}^{\infty} (g/\sigma_\theta^2) e^{-g^2/2\sigma_\theta^2} \, dg = 0.05 \qquad (2)$$

Solving the above equation for σ_θ we obtain

$$\sigma_\theta = \frac{10^{3.2}}{\sqrt{6}} \theta^{-2.5} = 647\theta^{-2.5} \qquad (3)$$

The interference power at the input of the earth station receiver due to the kth interferer is then given by

$$\begin{matrix}\text{Interference power} \\ \text{due to the kth interferer}\end{matrix} = \begin{Bmatrix}\text{Incident interference power at} \\ \text{earth station antenna}\end{Bmatrix} \times \begin{Bmatrix}\text{Gain of the} \\ \text{antenna at an} \\ \text{angle } (k\theta)\end{Bmatrix}$$

Hence the interference power is a stochastic variable which bears the statistical characteristics of the gain of the antenna. It also depends on the distribution of the SCPC carriers in the wanted channel bandwidth. The total interference power will then be the sum of a number of random quantities. It is assumed in the analysis that follows that all interfering sources are statistically independent.

III. DEVELOPMENT OF THE INTERFERENCE FORMULA

Consider the problem of an arbitrary narrowband signal interfering with an angle modulated carrier with arbitrary modulation. The desired and interfering signals are characterized by,

$$e_w(t) = \mathrm{Re}\{A \exp j(\omega_o t + x(t))\} \qquad (4)$$

$$e_k(t) = \mathrm{Re}\{A_k \exp j(\omega_k t + y_k(t) + \mu)\} \qquad (5)$$

respectively. In eqs. (4) and (5) A is the amplitude of the wanted signal, ω_o its angular frequency and x(t) its modulating signal; A_k is the amplitude of the kth interferer, ω_k its angular frequency and $y_k(t)$ its modulating signal. The amplitudes A and A_k are referred to the input of the satellite antennas.

It can be shown [3] that the baseband interference power spectrum for a low interference case (i.e., the power of the interfering signals is much smaller than the power of the wanted signal) is given by

$$I_{wk}(f) = \frac{1}{4}\left(\frac{A_k}{A}\right)^2 \left(S_{Ik}\left(f-f_d^k\right) * S_w(f) + S_{Ik}\left(f-f_d^k\right) * S_w(-f)\right) \qquad (6)$$

where

$\left(\frac{A_k}{A}\right)^2$ = Ratio of the power of the k^{th} interfering signal over the power of the wanted signal

* Denotes the convolution operator.

$f_d^k = f_o - f_k$, carrier frequency separation between interfering and wanted signals,

$S_{Ik}(f)$ = Low-pass equivalent power spectrum of the interfering RF signal,

$S_w(f)$ = Low-pass equivalent power spectrum of the wanted RF signal.

Consider a single interfering SCPC carrier from system k with $f_d{}^k = f_{d1}^k$ and assume

$$S_{Ik}(f) = \delta(f) \tag{7}$$

The interference power spectrum $I_{wk}(f)$ obtained by substituting (7) into (6) is given by

$$I_{wk}(f) = \frac{1}{4}\left(\frac{A_k}{A}\right)^2 \left[S_w(f-f_{d1}{}^k) + S_w(-f-f_{d1}{}^k)\right] \tag{8}$$

The interference power in a bandwidth Δf centered at $f=f_o$ can be approximated by

$$P_{1k}(f) = \frac{1}{4}\left(\frac{A_k}{A}\right)^2 \Delta f \left[S_w(f_o-f_{d1}{}^k) + S_w(-f_o-f_{d1}{}^k)\right] \tag{9}$$

If there are n interfering SCPC carriers with equal amplitudes and separations f_{d1}^k, f_{d2}^k ... f_{dn}^k from system k into the wanted system then the total interference power $P_k(f)$ is given by,

$$P_k(f) = \frac{1}{4}\left(\frac{A_k}{A}\right)^2 \Delta f \sum_{\ell=1}^{n} \left[S_w(f_o - f_{d\ell}^{\,k}) + S_w(-f_o - f_{d\ell}^{\,k}) \right] \tag{10}$$

For the configuration of Figure 1, the down-link interference power at the receiving earth station antenna can be calculated as follows. The carrier to interference ratio, C/I, at the input of the earth station receiver is given by

$$(C/I)_k = \frac{(A^2/2) G_{we} G_{ws}}{(A_k^2/2) G_{Ie}^k G_{Is}^k} \tag{11}$$

where

G_{we} = on-axis gain of wanted earth station antenna.

G_{ws} = gain of wanted satellite antenna in the direction of wanted earth station.

G_{Ie}^k = gain of the wanted earth station antenna in the direction of the k^{th} interfering satellite.

G_{Is}^k = gain of the kth interfering satellite antenna in the direction of the wanted earth station.

To simplify the problem it is assumed that the only statistical quantity of the expression (11) is G^k_{Ie}. Additional gain/loss between the interfering and wanted systems accounting for polarization, path loss differences and propagation effects are considered negligible.

We can write (11) in a simpler form

$$\left(\frac{C}{I}\right)^{-1}_k = A_o G^k_{ie} \tag{12}$$

where A_o is a constant given by

$$A_o = \frac{(A_k^2/2) G^k_{Is}}{(A^2/2) G_{we} G_{ws}} \tag{13}$$

The total interference power into the wanted earth station receiver is the sum of the individual interference powers weighted with the appropriate satellite and earth station antenna gains.

$$P_{total} = \sum_{k=-M}^{M} P_k(f) \left(\frac{G^k_{Is} G^k_{Ie}}{G_{we} G_{ws}}\right) \tag{14}$$

or

$$P_{total} = \sum_{k=-M}^{M} \frac{1}{4} A_o G^k_{ie} \left\{ \sum_{\ell=1}^{n} \left[S_w(f_o - f^k_{d\ell}) + S_w(f_o - f^k_{d\ell}) \right] \right\} \tag{15}$$

In eqs. (14) and (15) it is assumed that there are M symmetrically placed interfering satellites on each side of the desired satellite in the geostationary orbit. Since the k^{th} and the -kth interferers are assumed to contribute equally into the wanted signal equation (15) can be written

$$P_{total} = \sum_{k=1}^{M} \frac{1}{2} A_o G_{ie}^k \sum_{\ell=1}^{n} \left[S_w(f_o - f_{d\ell}^k) + S_w(-f_o - f_{d\ell}^k) \right] \qquad (16)$$

If we further make the definition

$$S_k \equiv \frac{1}{2} A_o \sum_{\ell=1}^{n} \left[S_w(f_o - f_{d\ell}^k) + S_w(-f_o - f_{d\ell}^k) \right] \qquad (17)$$

then equation (16) becomes

$$P_{total} = \sum_{k=1}^{M} G_{ie}^k S_k \qquad (18)$$

We observe that the total interference power is a stochastic quantity being the sum of the product of the random variables G_{ie}^k and S_k. A Gaussian approximation of the normalized wide-band wanted spectrum results in following simplification.

$$S_w(f) = \frac{1}{\sqrt{2\pi}\, f_\sigma} e^{-f^2/2f_\sigma^2} \qquad (19)$$

where

$$f_\sigma = m f_m \qquad (20)$$

m is the RMS modulation index and f_m the maximum baseband frequency of the wideband signal. Equations (16) and (17) become

$$P_{total} = \sum_{k=1}^{M} \frac{1}{2} A_o G_{ie}^k \sum_{\ell=1}^{n} \left\{ \exp\left[-\left(f_o - f_{d\ell}^k\right)^2 / 2f_\sigma^2\right] + \exp\left[-\left(f_o + f_{d\ell}^k\right)^2 / 2f_\sigma^2\right] \right\} \tag{21}$$

$$S_k = \frac{1}{2} A_o \sum_{\ell=1}^{n} \left\{ \exp\left[-\left(f_o - f_{d\ell}^k\right)^2 / 2f_\sigma^2\right] + \exp\left[-\left(f_o + f_{d\ell}^k\right)^2 / 2f_\sigma^2\right] \right\} \tag{22}$$

For a more statistically manageable analysis of this stochastic quantity, we need to determine its cumulative probability distribution function. To do this we proceed as follows: Let

$$y_j \equiv G_{ie}^j \; S_j \tag{23}$$

then equation (18) becomes

$$P_{total} = \sum_{j=1}^{M} y_j \tag{24}$$

The procedure is based on first determining the conditional cumulative probability distribution of P_{total} for $S_j = s_j$. In Appendix A, a method of determining the conditional probability density function $f(z/S_1=s_1, S_2=s_2, \ldots)$ of the stochastic variable P_{total} is outlined. Using that density function we can write:

$$\text{Prob}\left[P_{total} \geq P_0 / S_1 = s_1,\ S_2 = s_2, \ldots,\ S_M = s_M\right]$$

$$= \int_{P_0}^{\infty} f(z/S_1 = s_1,\ S_2 = s_2, \ldots,\ S_M = s_M)\,dz \qquad (25)$$

Since the interference power falls off rapidly with increasing separation θ we restrict the number of interferers to four, and write.

$$\text{Prob}\left[P_{total} \geq P_0 / S_1 = s_1,\ S_2 = s_2,\ S_3 = s_3,\ S_4 = s_4\right]$$

$$= \int_{P_0}^{\infty} f(x/S_1 = s_1,\ S_2 = s_2,\ S_3 = s_3,\ S_4 = s_4)\,dx \qquad (26)$$

The unconditional cumulative probability distribution function of the variable P_{total} can be determined from equation (26) using the equation

$$\text{Prob}\left[P_{total} \geq P_0\right] = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \text{Prob}(P_{total} \geq P_0 / S_1 = s_1,\ S_2 = s_2,\ S_3 = s_3,\ S_4 = s_4)$$

$$f_s(s_1, s_2, s_3, s_4)\,ds_1, ds_2, ds_3, ds_4 \qquad (27)$$

where $f_s(\cdot)$ is the joint probability density function of the variables S_1, S_2, S_3, S_4.

Since the probability density function $f_s(\cdot)$ is not known we use computer simulation to find an estimate of the expression given by (27).

The simulation procedure that was followed to obtain an estimate of Prob($P_{total}>P_o$) is described below:

Step 1: Obtain a random set of frequencies for a single realization of each of the four interfering systems under the constraints of the assignment procedure described in Section II.

Step 2: Calculate equation (26) for this realization.

Step 3: Repeat 1 and 2 until the sample mean of the Prob($P_{total}>P_o$) differs from the true mean by less than 0.1 dB with a probability of 0.99.

IV. NUMERICAL RESULTS

Numerical results were obtained for a specific system model. In this model it was assumed that each interfering satellite had 1600 frequency slots per 36 MHz transponder for the SCPC transmission but was constrained to operate with a maximum of 1000 carriers. The activity factor for this case and the details of the interfered-with FDM/FM spectrum are given in Appendix B. The power P_0 in Equation (26) is normalized with respect to P_o which is defined as the interference that would be obtained if all the SCPC frequency slots were active simultaneously and the antenna sidelobe characteristics matched the deterministic CCIR expression (32-25 log θ). Calculations were done for adjacent satellite separations of 5 degree, 4 degree, and 3 degree. The results are shown in Figure 2. The reference power P_o for all the three curves in Figure 2 is the reference power for 5 degree separation. The reference power P_o is the commonly used worst-case interference power in interference calculations.

V. DISCUSSION AND CONCLUSIONS

It has been shown that the statistical characteristics of the intersystem interference power of the satellite communication system under consideration can be divided into two independent parts. One part reflects the statistical nature of the SCPC carrier assignment procedure and the other part reflects the statistical nature of the sidelobe gain of the receiving antenna. It was assumed that the statistical behavior of the antenna sidelobe gain follows a Rayleigh distribution. The same type of distribution can be used to model other types of inherently statistical parameters of a given satellite system such as unintentional transmit power variations, antenna pointing errors and so on. For cases like these we need measured data to deduce the statistical behavior and more specifically the probability distribution of these parameters. From Figure 2, it is seen that for a typical set of interference conditions (5 degree satellite spacing) between a wideband transmission system and a multiple SCPC system the interference power is less, by 6 dB with a confidence level 99 percent, than the worst case-power assumed in intersystem interference calculations. Thus, ignoring the statistical nature of interference results in a substantially pessimistic estimation.

As expected the interference is very sensitive to orbital spacing. If we want to achieve at a 4 degree separation the same confidence level for the interference power, we have to accept 2.5 dB higher interference power than at a 5 degree separation but it is still 3.5 dB lower than the worst-case power. These type of considerations permit a more realistic trade off of interference power for orbit utilization and vice versa. The methodology presented in this paper is quite general, and as metnioned before, it can be applied to situations where other statistical parameters, which were ignored in this study, play an important role.

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HO-4375-$\frac{\text{SCM}}{\text{PS}}$-blb

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References
Appendix A
Appendix B
Figures 1 and 2

REFERENCES

1. C.C.I.R. Study Group, Period 1974-1976, Document 4/32E, (Japan) January 1976.
2. M. C. Jeruchim, A Statistical Approach to Satellite Interference Levels, 1978 International Conference on Communications Record, Vol. 3, pp. 35.3.1-35.3.4.
3. V. K. Prabhu and L. H. Enloe, Interchannel Interference Considerations in Angle-Modulated Systems, B.S.T.J., pp. 2333-2358, September 1969.
4. M. C. Jeruchim, A Survey of Interference Problems and Applications to Geostationary Satellite Networks. Proceedings of IEEE, pp. 317-331, March 1977.
5. B. A. Pontano, J. C. Fuenzalida, and N. K. M. Chitre, Interference into Angle-Modulated Systems Carrying Multichannel Telphony Signals, IEEE Transactions on Communications, Vol. COM-21, No. 6, pp. 714-727, June 1973.
6. A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill 1965.
7. K. Bullington and J. Fraser, Engineering Aspects of TASI, B.S.T.J. Vol. 38, 353-364, 1959.

APPENDIX A

The cumulative distribution function of the variable y_j is given by

$$\text{Prob}\left(y_j \le y/S_j = s_j\right) = \text{Prob}\left(G^j_{ie}\ S_j \le y/S_j = s_j\right) \qquad A.1$$

Since G^j_{ie} is Rayleigh distributed, we can write

$$\text{Prob}\left(y_j \le y/S_j = s_j\right) = \text{Prob}\left(G^j_{ie} \le \frac{y}{s_j} / S_j = s_j\right) \qquad A.2$$

or

$$\text{Prob}\left(y_j \le y/S_j = s_j\right) = \int_0^{y/s_j} \frac{g}{\sigma_\theta^2}\, e^{-g^2/2\sigma_\theta^2}\, dg \qquad A.3$$

$$= -e^{-g^2/2\sigma_\theta^2}\Big]_0^{y/s_j}$$

Hence

$$\text{Prob}\left(y_j \le y/S_j = s_j\right) = \left(-e^{-(y/s_j)^2/2\sigma_\theta^2} + 1\right) \qquad A.4$$

The conditional density function of y_j can be obtained from equation A.4 by taking the derivative of $\text{Prob}(y_j \le y/S_j = s_j)$ with respect to y, i.e.

$$f_{y_j}(y/S_j=s_j) \equiv \frac{d}{dy}\, \text{Prob}(y_j \le y/S_j=s_j) \qquad A.5$$

then

$$f_{y_j}(y/S_j=s_j) = \frac{d}{dy}\left[-e^{-(y/s_j)^2/2\sigma_\theta^2} + 1\right] = \frac{y}{s_j^2\sigma_\theta^2}\, e^{-(y/s_j)^2/2\sigma_\theta^2}$$

which is again a Rayleigh distribution. A.6

The probability density function of a sum of independent random variables is given by the convolution of the density functions of the individual random variables [7]. For the case of two Rayleigh distributed variables, the probability density of their sum is shown [2] to be given by the expression

$$f_{Z_1}(z/S_1=s_1, S_2=s_2) = \left[\frac{\alpha_1}{\sigma_{T12}}\right]^2 z\, e^{-z^2/2\sigma_{\theta 1}^2}$$

$$+ \left[\frac{\alpha_2}{\sigma_{T12}}\right]^2 z\, e^{-z^2/2\sigma_{\theta 2}^2} - \sqrt{\frac{\pi}{2}}\,\frac{\sigma_{\theta 1}\sigma_{\theta 2}}{\sigma_{T12}^3}\left[1 - \frac{z^2}{\sigma_{T12}^2}\right] e^{-z^2/2\sigma_{T12}^2}$$

$$\left\{\operatorname{erf}(z\sigma_{\theta 2}/\sqrt{2}\,\sigma_{\theta 1}\sigma_{T12}) + \operatorname{erf}(z\sigma_{\theta 1}/\sqrt{2}\,\sigma_{\theta 2}\sigma_{T12})\right\} \qquad \text{A.7}$$

where

$$Z_1 = y_1 + y_2\,,\ \sigma_{\theta 1}^2 = (647)^2\theta_1^{-5} s_1^2,\ \sigma_{\theta 2}^2 = (647)^2\theta_2^{-5} s_2^2$$

$$\sigma_{T12}^2 = \sigma_{\theta 1}^2 + \sigma_{\theta 2}^2,\ \alpha_1^2 = \frac{\sigma_{\theta 1}^2}{\sigma_{T12}^2},\ \alpha_2^2 = \frac{\sigma_{\theta 2}^2}{\sigma_{T12}^2} \qquad \text{A.8}$$

For the case of four Rayleigh distributed random variables, the probability density of their sum can be expressed by

Let $$\tilde{Z} = y_1 + y_2 + y_3 + y_4 \qquad \text{A.9}$$

then $$\tilde{Z} = Z_1 + Z_2 \qquad \text{A.10}$$

where $$Z_1 = y_1 + y_2 \quad \text{and} \quad Z_2 = y_3 + y_4 \qquad \text{A.11}$$

Similarly for the random variables y_2, y_3, the probability density of their sum is given by

$$f_{Z_2}(z/S_3=s_3, S_4=s_4) = \frac{\alpha_3^2}{\sigma_{T34}^2} z\, e^{-z^2/2\sigma_{\theta 3}^2} + \frac{\alpha_4^2}{\sigma_{T34}^2} z\, e^{-z^2/2\sigma_{\theta 4}^2}$$

$$- \sqrt{\frac{\pi}{2}} \frac{\sigma_{\theta 3}\sigma_{\theta 4}}{\sigma_{T34}^3} \left(1 - \frac{z^2}{\sigma_{T34}^2}\right) e^{-z^2/2\sigma_{T34}^2} \left[\mathrm{erf}(z\sigma_{\theta 3}/\sqrt{2}\,\sigma_{\theta 3}\sigma_{T34}) + \mathrm{erf}(z\sigma_{\theta 4}/\sqrt{2}\,\sigma_{\theta 4}\sigma_{T34})\right] \qquad \text{A.12}$$

where

$$\left.\begin{aligned} &Z_2 = y_3 + y_4 \;, \; \sigma_{\theta 3}^2 = (647)^2 \theta_3^{-5} s_3^2, \; \sigma_{\theta 4}^2 = (647)^2 \theta_4^{-5} s_4^2 \\ &\sigma_{T34}^2 = \sigma_{\theta 3}^2 + \sigma_{\theta 4}^2 \;, \; \alpha_3^2 = \frac{\sigma_{\theta 3}^2}{\sigma_{T34}^2} \;, \; \alpha_4^2 = \frac{\sigma_{\theta 4}^2}{\sigma_{T34}^2} \;. \end{aligned}\right\} \text{A.13}$$

Each of the variables Z_1 and Z_2 has a probability density function of the form given by A.7. The probability density function of $\tilde{Z}$ then can be given by

$$f_{\tilde{Z}}(\tilde{z}/S_1=s_1,S_2=s_2,S_3=s_3,S_4=s_4)$$

$$= \int_0^{\tilde{z}} f_{Z_1}(x/S_1=s_1,S_2=s_2)f_{Z_2}(z-x/S_3=s_3,S_4=s_4)dx \qquad A.14$$

where

$f_{Z_1}(\cdot)$ and $f_{Z_2}(\cdot)$ are given by equation A.7

This procedure can be extended to any number of variables.

APPENDIX B

CALCULATION OF $\overline{S}_k$

At a busy hour a maximum of N talkers are assumed to be conversing. The probability that at any given time the number of talkers exceed N_o is given by

$$B(N_o,N,P) = \sum_{x=N_o}^{N} \frac{N!}{x!(N-x)!} p^x(1-p)^{N-x} \qquad \text{B.1}$$

where p is the activity factor of each talker. The cumulative binomial distribution of B.1 can be approximated very well by the following normal distribution

$$B(N_o,N,p) \cong \frac{1}{\sqrt{2\pi}} \int_{u=y}^{\infty} e^{-u^2/2}\, du \qquad \text{B.2}$$

where

$$y = (N_o - N_p - 1/2)/\sqrt{np(1-p)} \qquad \text{B.3}$$

From tables of normal distribution we see that y=1.29 for $B(N_o,N,p) = 0.1$ and y=2.33 for $B(N_o,N,p) = 0.01$. It is assumed that for each SCPC system the maximum number of talkers is 1000. Substituting N=1000 and p=0.4 into B.3 we obtain

$$N_o = 430 \text{ for } B(N_o, N, P) = 0.01$$

Each SCPC system has 1600 channel slots and we define the system activity factor by the ratio 430/1600 = 0.27. The wideband wanted channel is chosen to be an FDM/FM with the following parameters

No. of channels = 72
Top baseband frequency = 204.0 kHz
RF bandwidth = 2.50 MHz
RMS deviation for 0 dBm0 test tone = 125.0 kHz
Peak factor = 12.0 dB

It is further assumed that the FM/FDM spectrum is centered at the midfrequency of the transponder. The quantity f_σ is computed from the RMS deviation given above for 0 dBm0 test tone and the average load P_{av} given by

$$P_{av} = -1 + 4 \log N (\text{dBm0}). \qquad \text{(B.4)}$$

The value of f_σ turned out to be 262.05 kHz.

The SCPC channel slots were assumed to be 22.5 kHz apart (no half spacing or staggering) and the pairing was done by choosing the first frequency at random and the second one 18 MHz away from the first one (β = 800 in II(a)).

The following value was obtained for the reference case where all the SCPC channel slots are assumed to be active simultaneously. No. of SCPC carriers N_{cr} within the

FM/FDM bandwidth = 111

Value of S_k per eq. (22) = $14.6a_o$ B.5

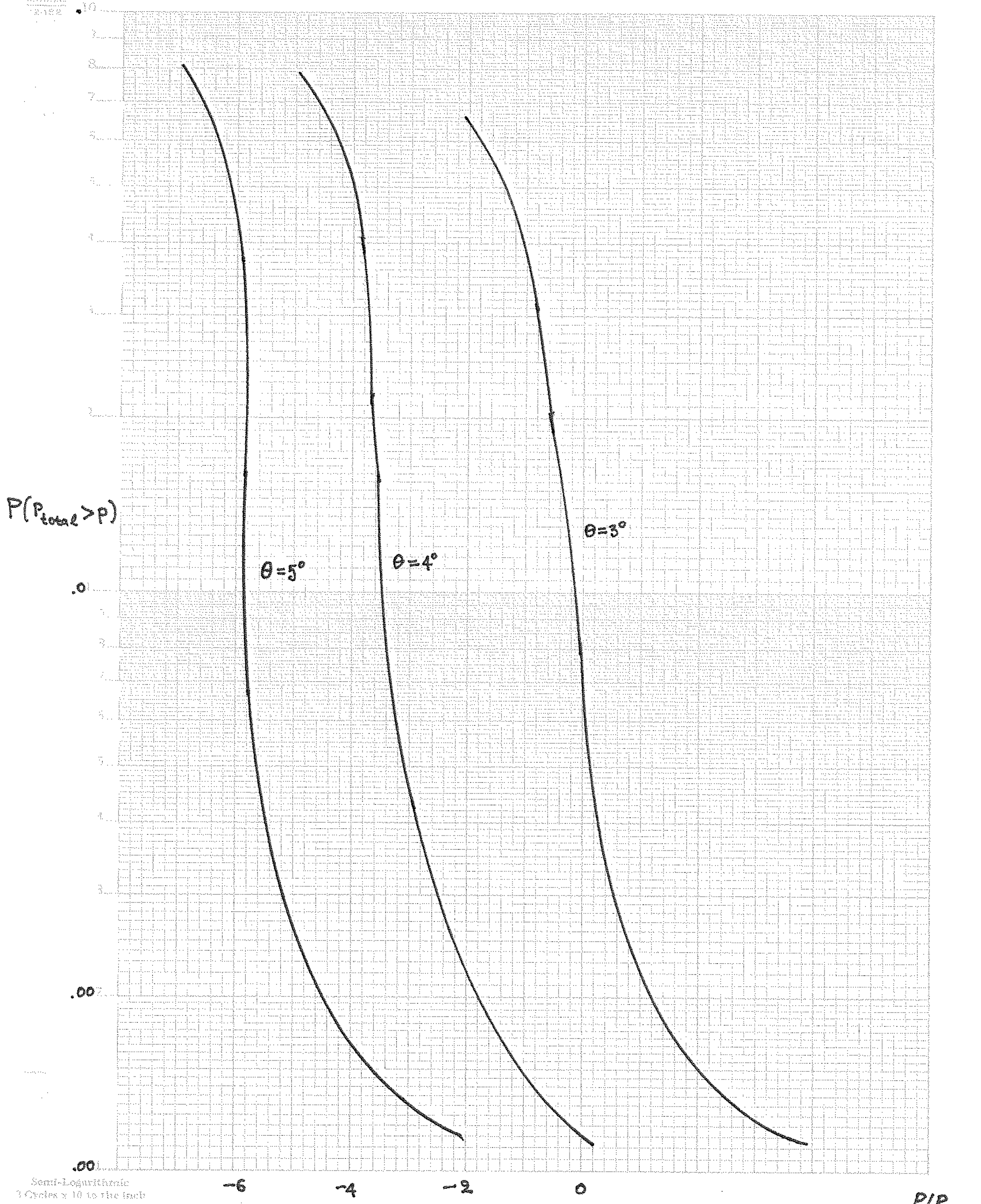


FIGURE 2 PROBABILITY VS RELATIVE INTERFERENCE POWER

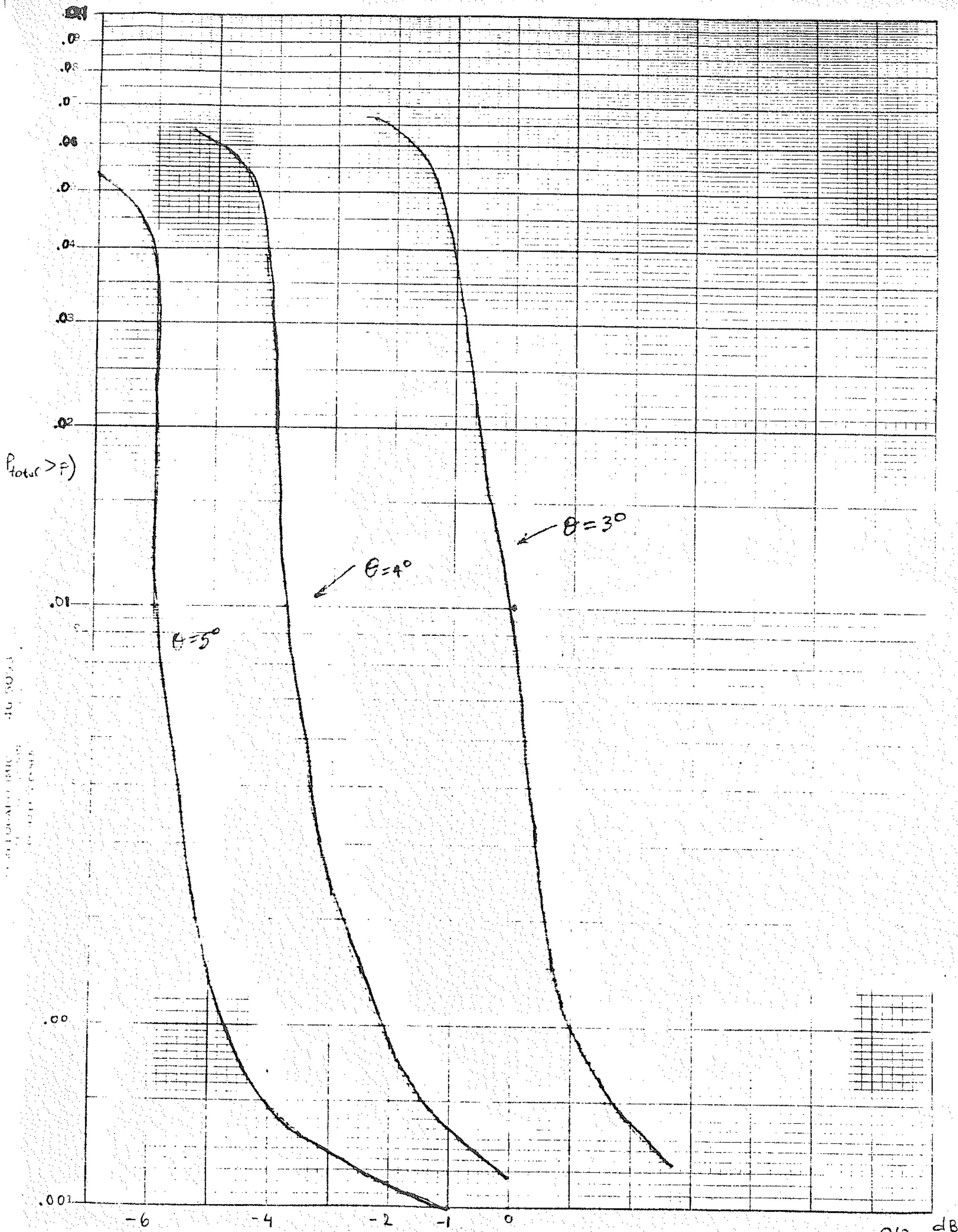


FIGURE 2. PROBABILITY VS RELATIVE INTERFERENCE POWER

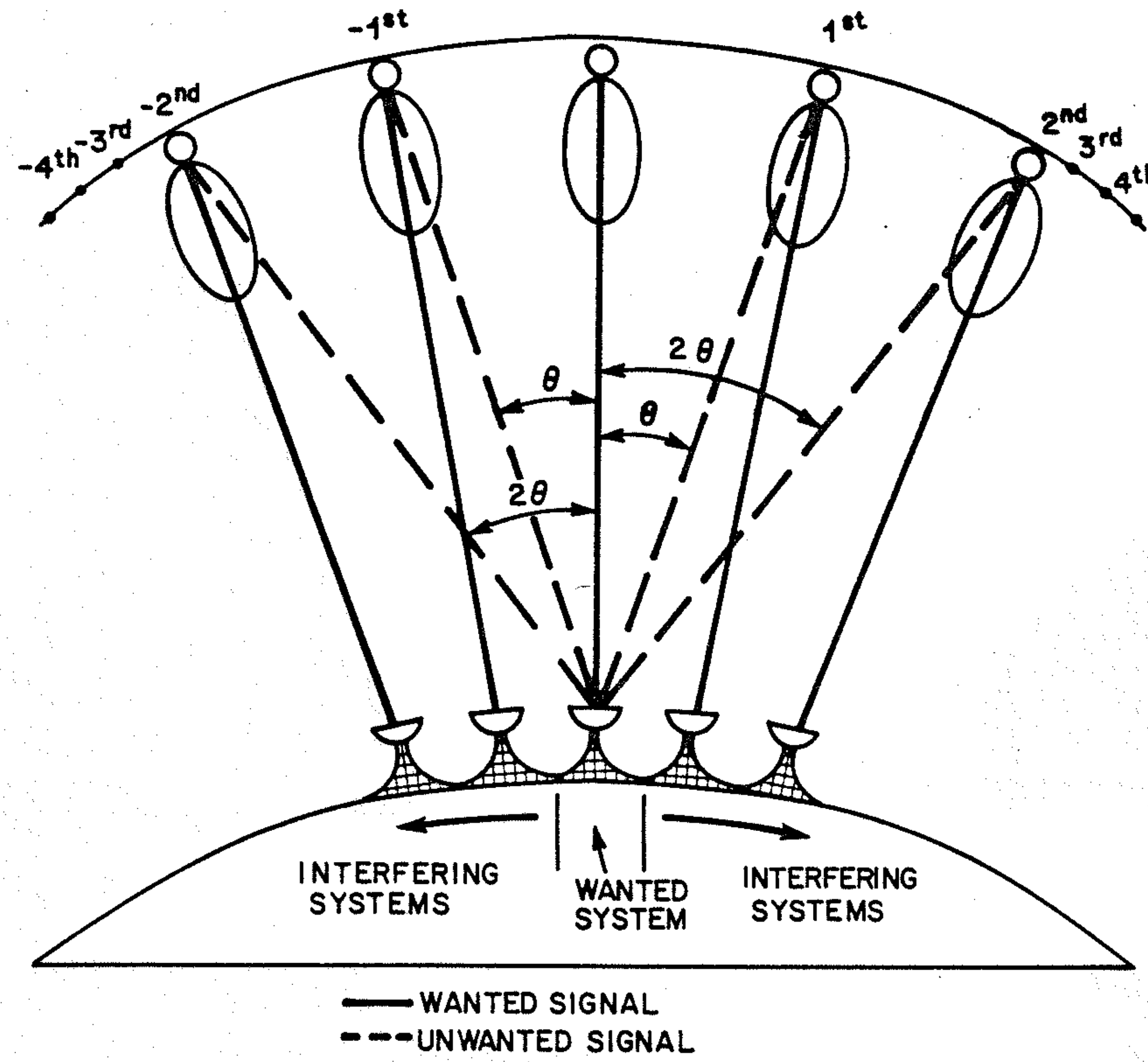
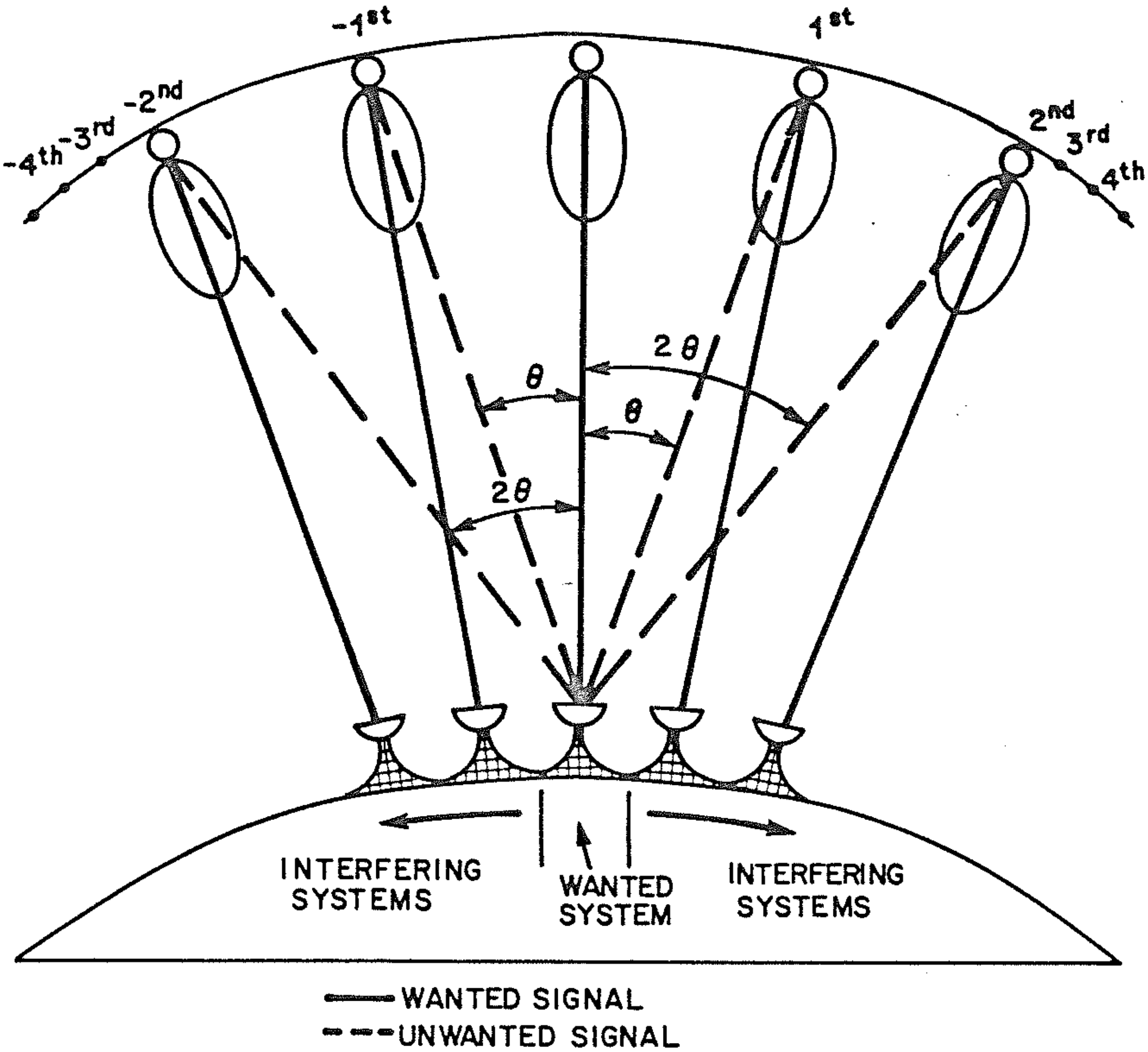
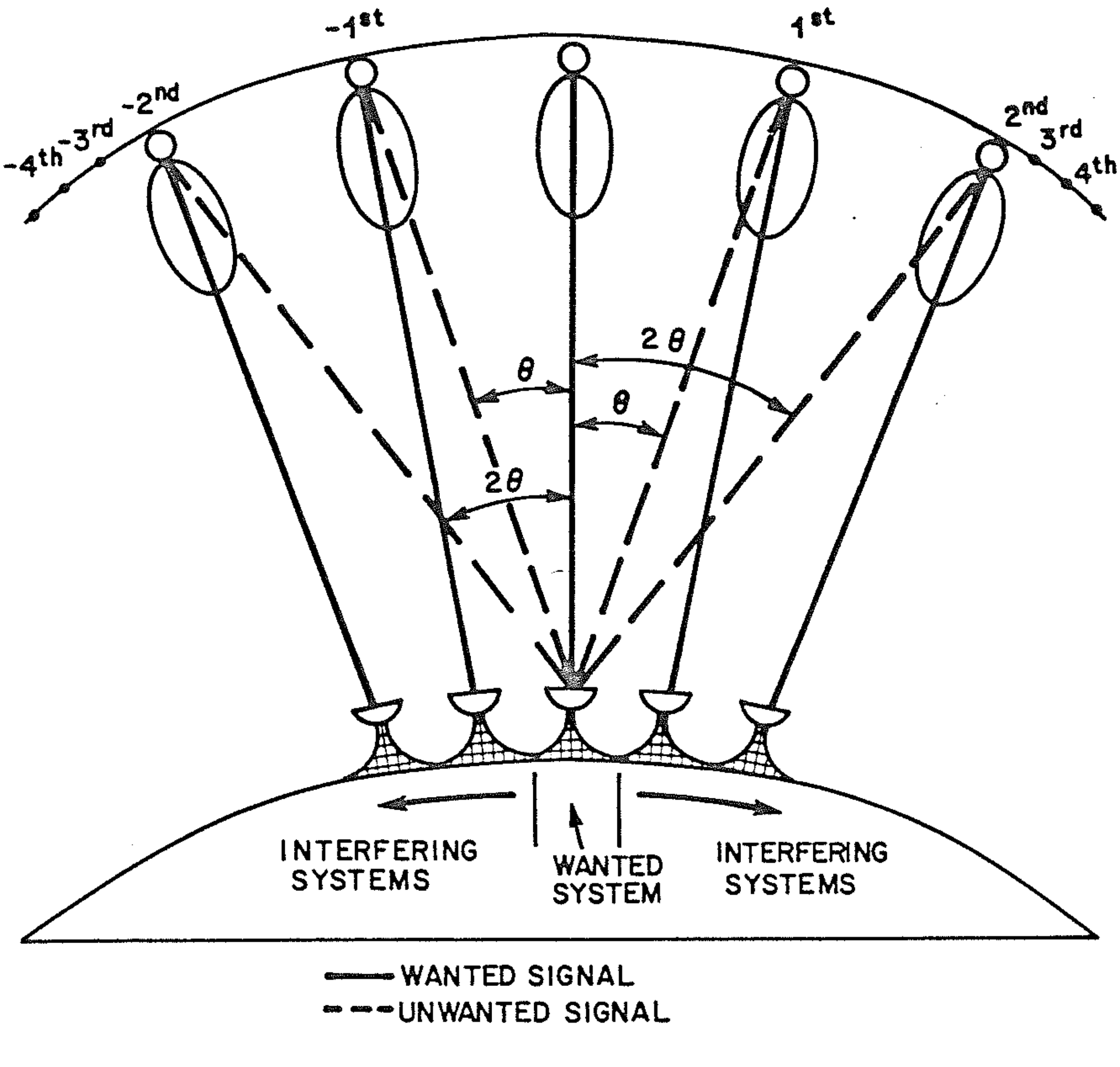


FIGURE 1 INTERFERENCE MODEL


-1st
1st
-4th -3rd -2nd
2nd
3rd
4th
2θ
θ
θ
2θ
INTERFERING SYSTEMS
WANTED SYSTEM
INTERFERING SYSTEMS
WANTED SIGNAL
UNWANTED SIGNAL


-1st
1st
-4th -3rd -2nd
2nd 3rd 4th
θ
2θ
θ
2θ
INTERFERING SYSTEMS
WANTED SYSTEM
INTERFERING SYSTEMS
WANTED SIGNAL
UNWANTED SIGNAL