Oscillator Phase Noise:
A 50-year Retrospective

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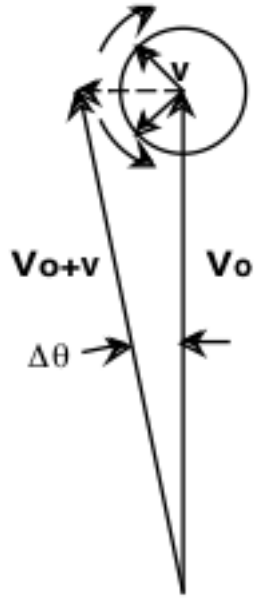
Some Preliminary Generalities

- **Communications: Energy efficient**
- **Experiment trumps theory**
  - **Good theory predicts results**
- **The model is not the thing**
  - **Linear model won't show nonlinear effects**
  - **Perturbation effects can be linear**
- **What has happened in 50 years?**
  - **Moore's law: IC's, PC's, iPhones, Internet, WiFi**
  - **California Microwave 1968-1993, Stanford 1994-**
- **Standards come before wide acceptance**
  - **What can be learned from my experiences?**

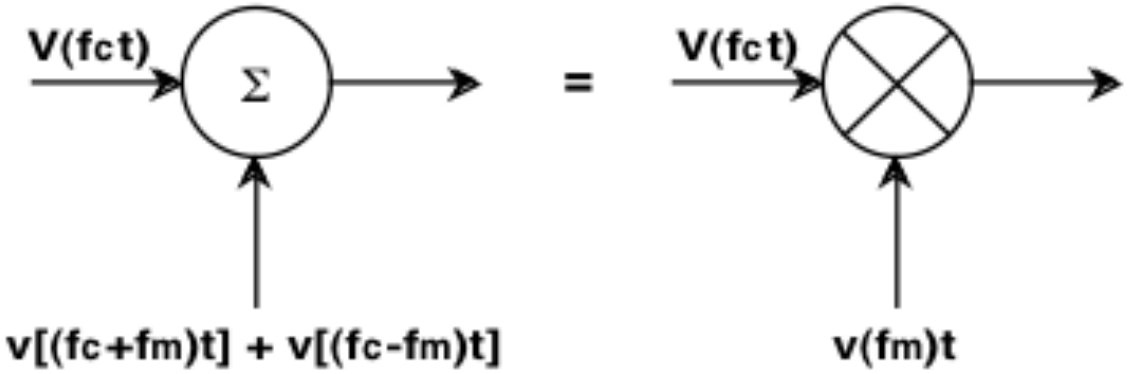
Review: Oscillator Phase Noise

- **Ideal: $v(t) = v\cos\omega t$**
- **Actual: $v(t) = v\cos\{\omega t + \phi(t)\}$**
  - **$\phi(t)$ from random noise**
  - **$\phi(t)$: Output of a phase detector**
  - **$d\phi/dt$: FM**
- **All oscillators are nonlinear**
  - **Some form of limiting**
  - **AM<<PM, especially if multiplied x N**
- **Small noise perturbations linear [Penfield 1966]**
- **Derive oscillator noise from circuit parameters?**



Additive, Modulative Equivalent

- **Equivalence of additive, modulative forms**
  - **Additive dependent on P_s/N**
    - **Cascaded stages: Divide by gain (Friis law)**
  - **Modulative independent of P_s**
    - **Cascaded stages: Adds with no reduction by gain**

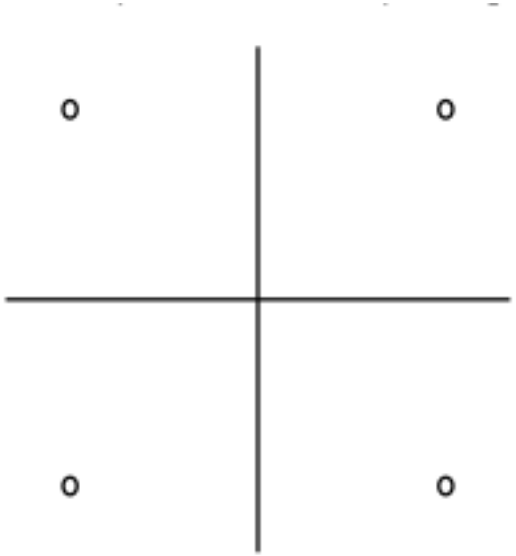


Phase Noise in Different Systems

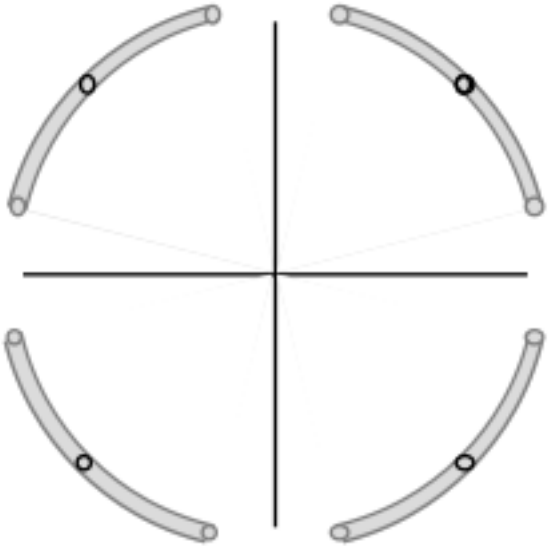
- **Phase modulated systems — digital QAM**
  - **Jitter = Integrated phase**
- **Dynamic range limitation**
  - **Coherent radar: Ground signal (clutter)**
  - **Cellular systems: Near-far problem**
  - **Reciprocal mixing in receivers**
- **Precision time and frequency measurement**
  - **Time domain variances**
  - **Frequency domain spectrum**
  - **Fourier relationship**

Phase Modulated Systems

- **QPSK constellation example**



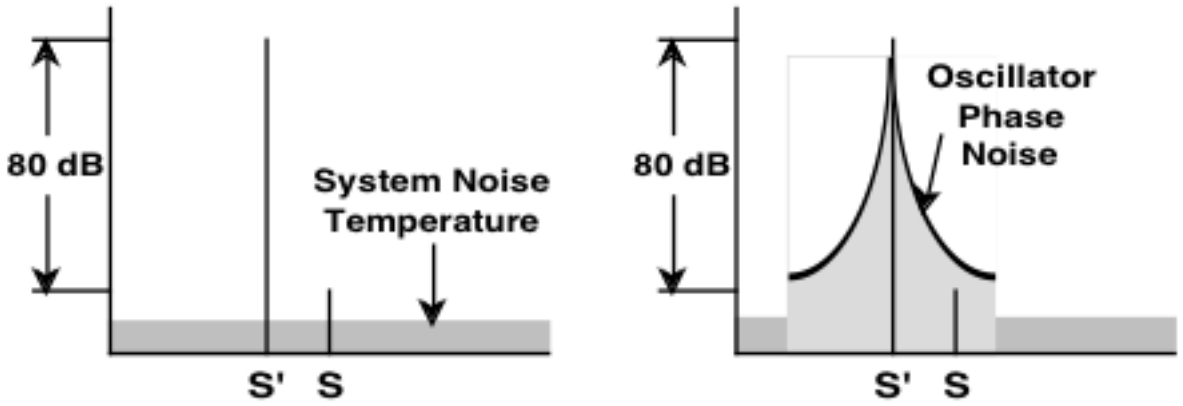
Noise-free



Phase noise creates uncertainty

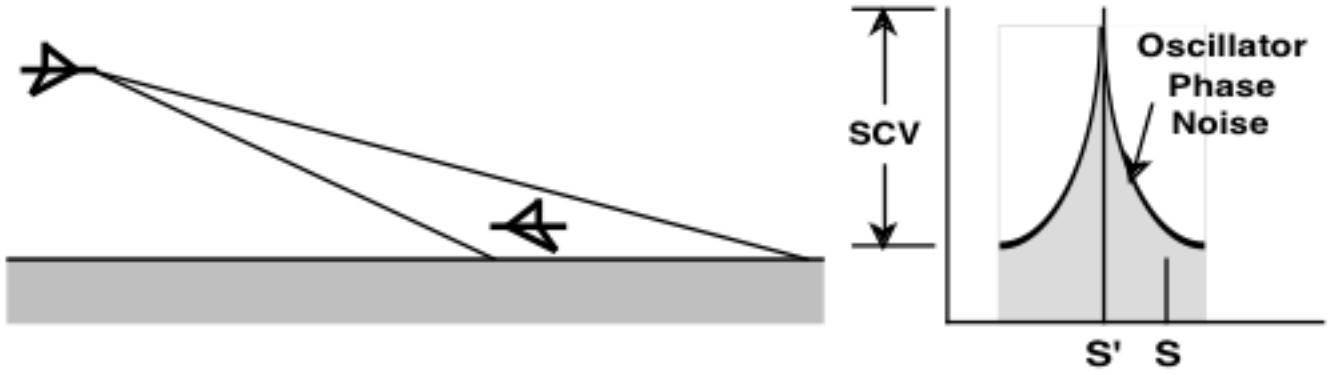
Phase Noise on Adjacent Signal

- **Large signal with phase noise masks smaller one**
  - **“Near-far problem”**



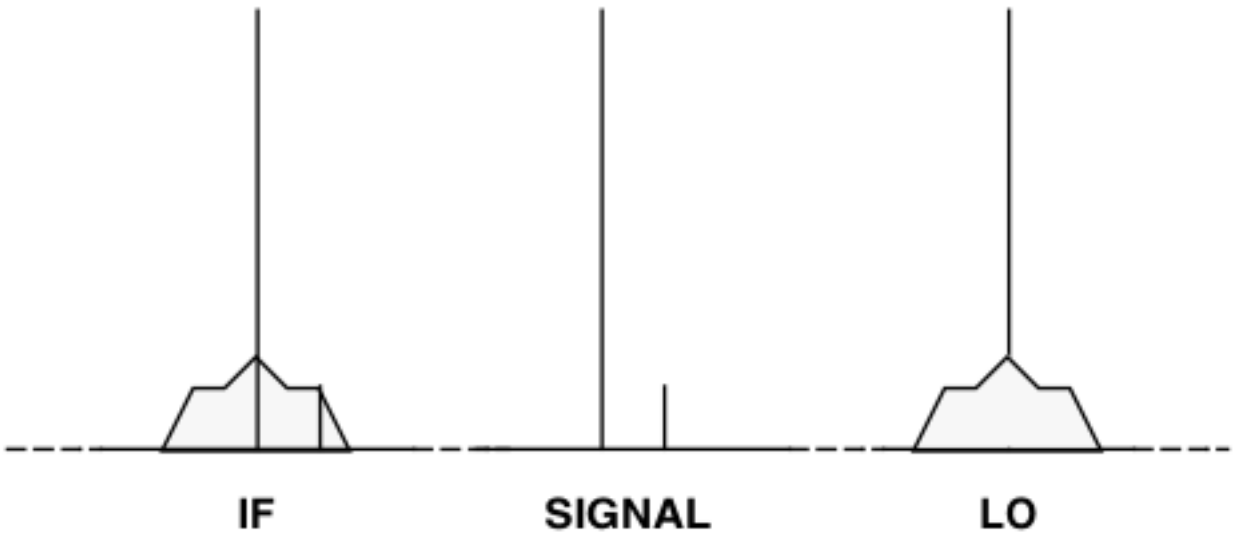
Doppler Radar: Subclutter Visibility

- **Ground clutter: Phase noise masks targets**
  - **Δf = 10 kHz/Mach number @ 10 GHz**



Reciprocal Mixing in Receivers

- **LO noise mixed onto clean signals**
  - **Masks weaker signals, even if stronger is clean**



Today: Measures of Stability

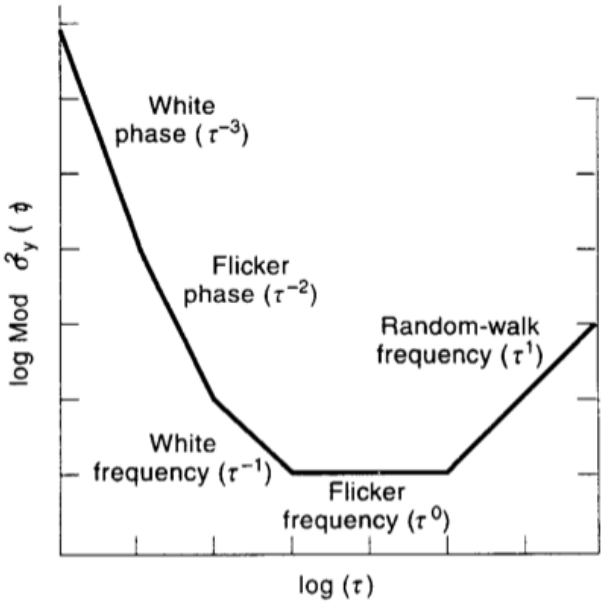
- **Time domain: Allan Variance $\sigma_y(\tau)$**
  - $\sigma_y(\tau) = \sum_{\mu=-2}^{1} h_\mu \tau^\mu$
- **Frequency Domain: Spectral density $S_\phi(f)$**
  - $S_\phi(f) = \sum_{\beta=-4}^{0} b_\beta f^\beta$

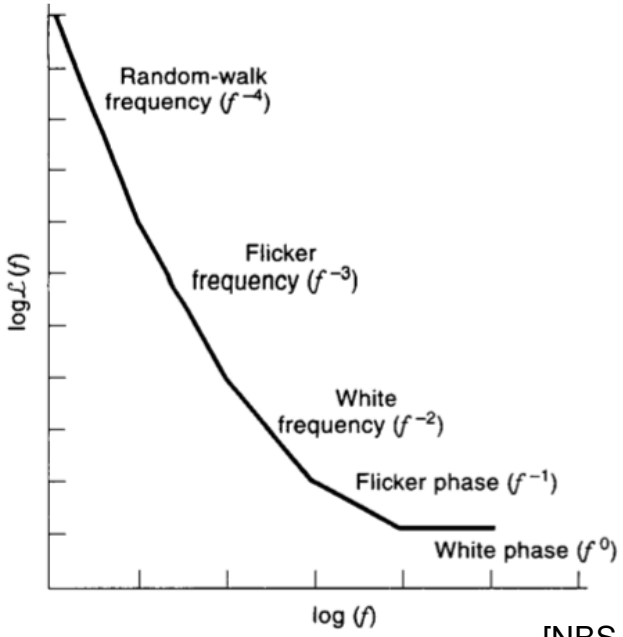
	β	μ
White PM	0	-2
Flicker PM	-1	-2
White FM	-2	-1
Flicker FM	-3	0
Ran. Walk FM	-4	1

Today: Graphic Definitions, Power Law

- **Allan Variance (mod)**
- **Power Spectral Density**

$$\text{mod}\ \sigma_y(\tau) = \sum_{\mu=-3}^{1} h_\mu \tau^\mu \quad \Leftrightarrow \quad S_\phi(f) = \sum_{\beta=-4}^{0} b_\beta f^\beta$$

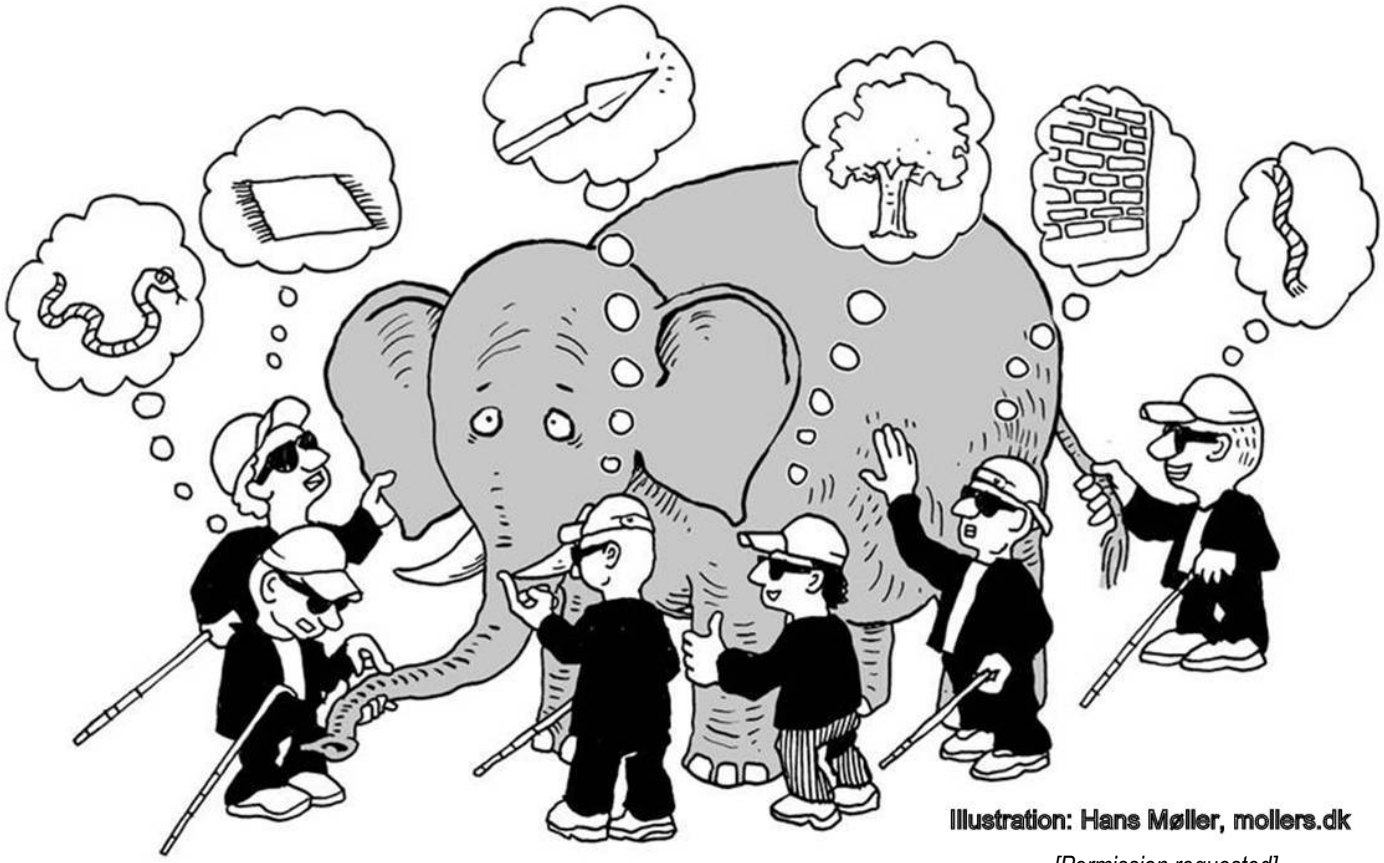




[NBS 1065]

17,000 publications on Phase Noise & Frequency Stability

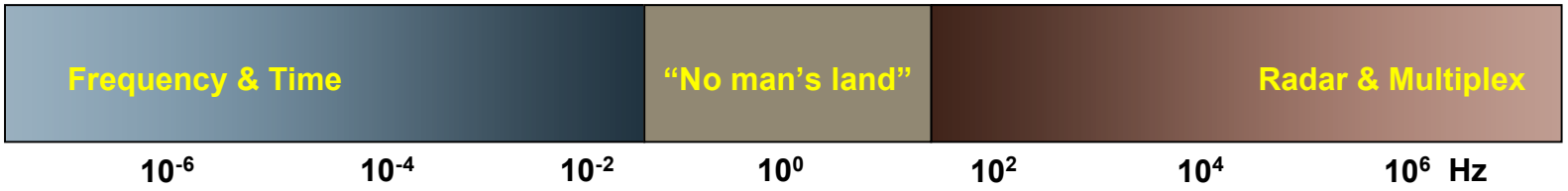
50 Years Ago …

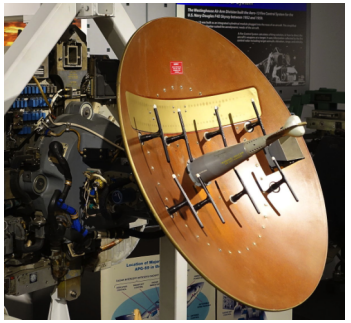


[Permission requested]

50 Years Ago: The Two "Guilds"

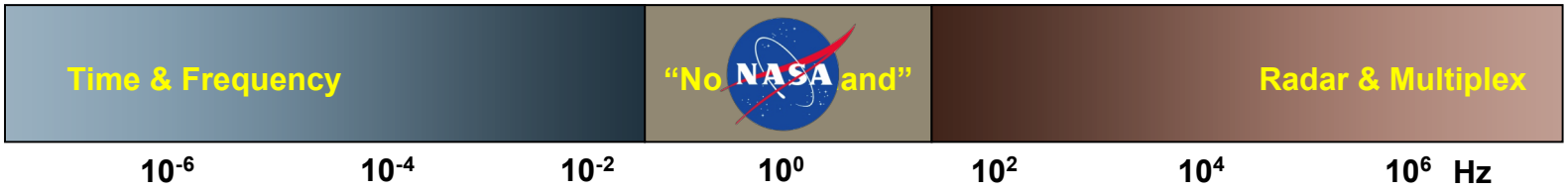
- **Frequency and Time**
  - **10^{-N} per decade**
  - **Focus: Stability vs. Time**
- **Airborne Doppler Radar**
  - **Mach 1 ≈ 10 kHz @ 10 GHz**
- **Frequency Division Multiplex**
  - **60-108 kHz & up**

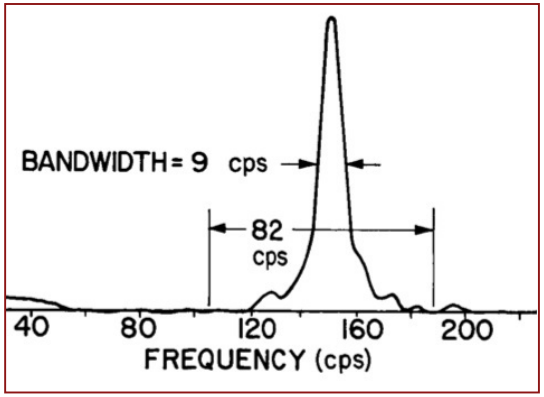




Space Program: New Requirements

- **NASA into "No Man's Land"**
  - **Narrowband phase lock loops, digital modulation**
- **Meld definitions with conversion**
  - **Frequency- & time-domain**





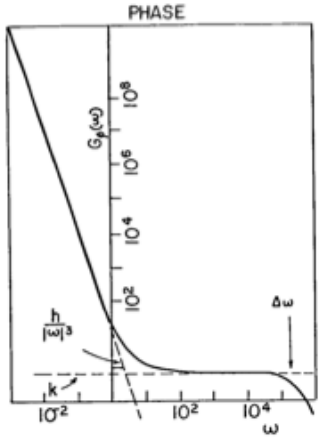
[Sydnor 1964]

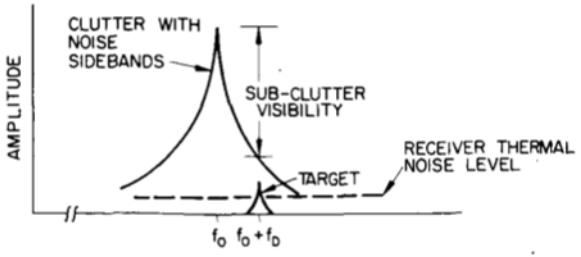
IEEE Standards Committee 14.7

- **Unify definitions of frequency stability**
- **Objective: A standard**
- **1964 — IEEE-NASA Symposium**
- **1966 — Special Issue *Proc IEEE***
- **1971 — Barnes, et al, "Characterization…"**
- **1988 — IEEE Std 1139 Rev 1**

1964 IEEE-NASA Symposium

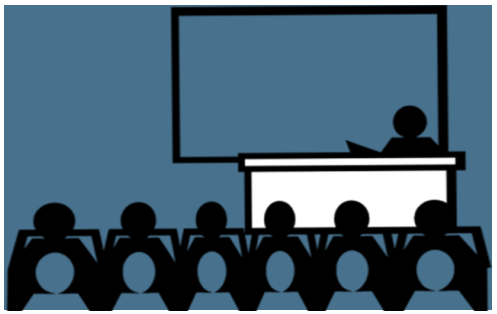
- **350 Attended**
- **Papers from time, space, radar**
- **Panels: Rigor vs. practice, “leave us out?”**

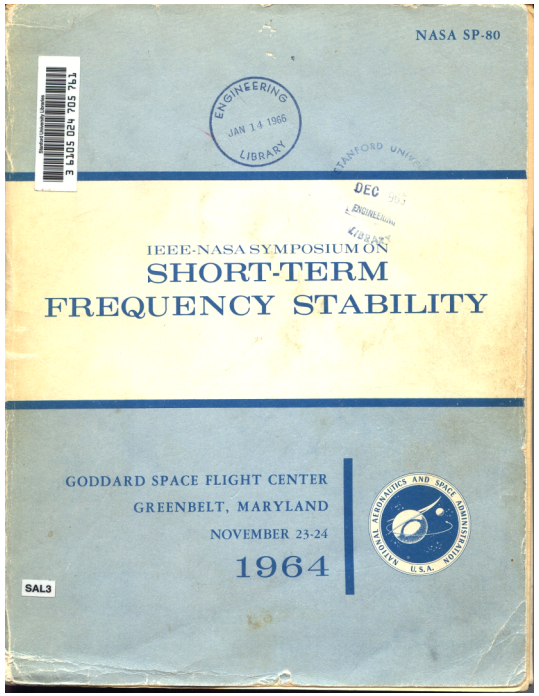




[Leeson-Johnson 1964]

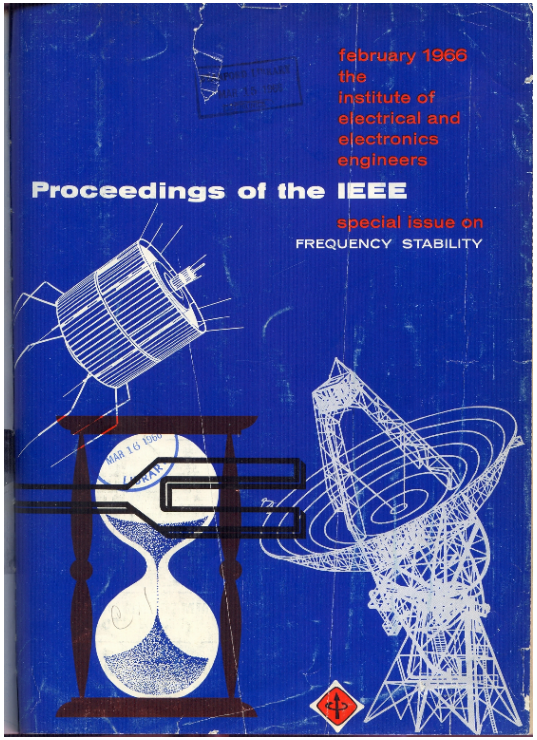
[Barnes-Allan 1964]





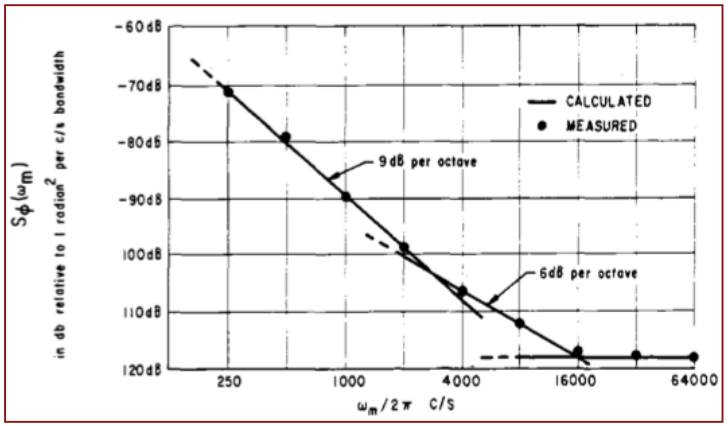
1966 Special Issue: Frequency Stability

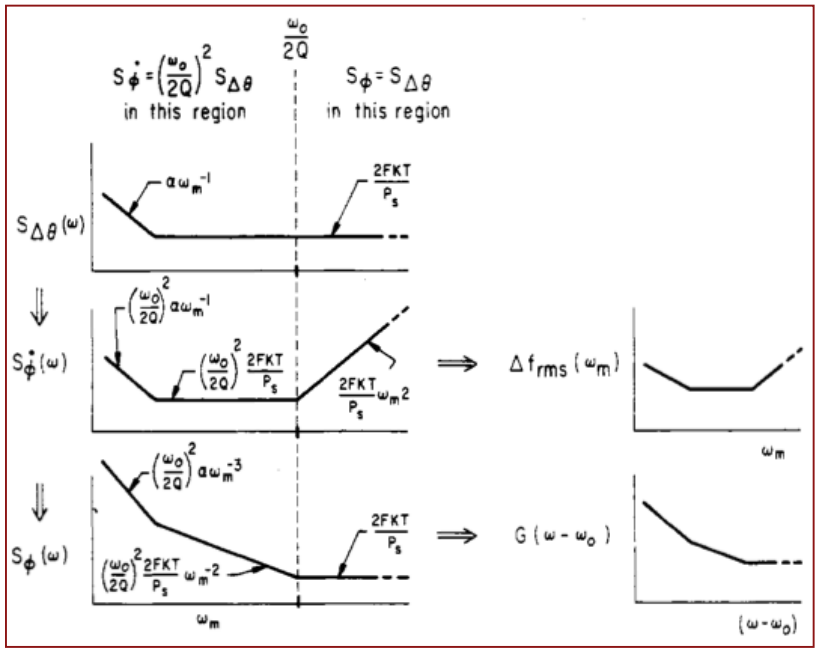
- **Committee as editors & authors**
- **Allan Variance paper** [Allan 1966]
  - $\sigma_y(\tau) \Leftrightarrow S_\phi(f)$
- $S_\phi(f)$ **from** F, Q, P_s**?**
  - **Meld prior, new knowledge**
  - **2 pages, one month to go**



Phase Noise Model 1966

- **$S_\phi(f)$: Multi-segment for single resonator**
  - $f < f_o/2Q$: $(\Delta f/f_o)^2 \approx kf^{-0}$ [Edson 1960, Mullen 1960]
  - $f > f_o/2Q$: $\mathcal{L}(f) \approx FKT/P_s$ [Leeson-Johnson 1964]
  - $|H(f)|^2 = 1 + (f_o/2Qf)^2$ [Leeson 1966]
- **$S_\phi(f) = |H(f)|^2 * S_{\Delta\theta}(f)$**
  - $S_{\Delta\theta}(f) = \alpha/f + FKT/P_s$

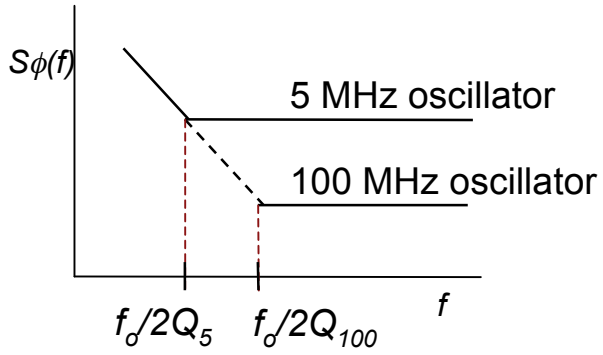


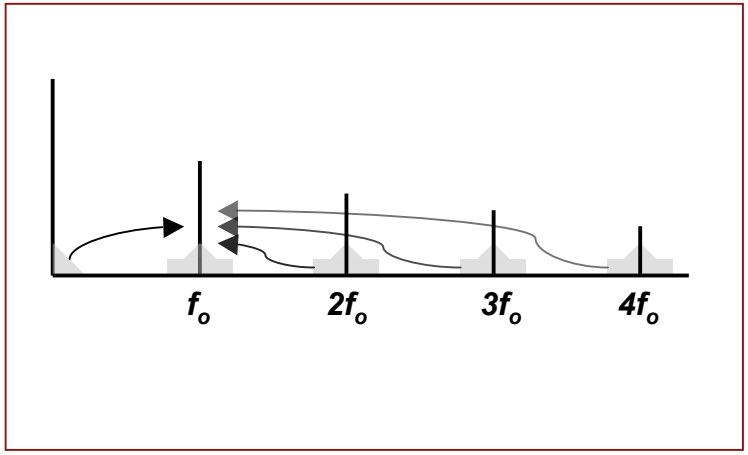


Notes in Phase Noise Model 1966

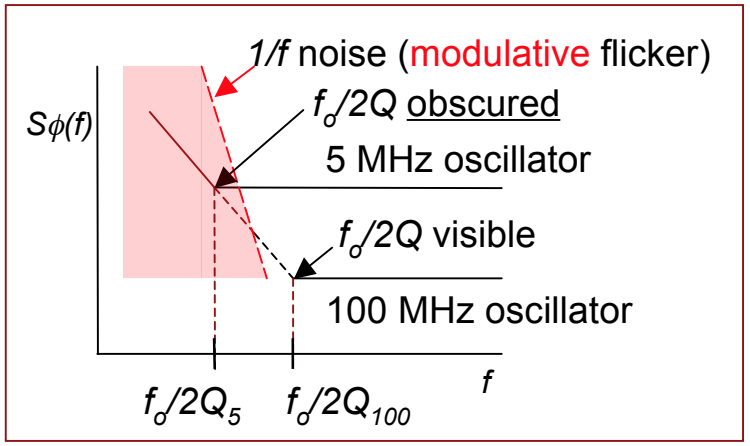
- **Harmonic noise mixed**
  - ***1/f* even if linear**
- **Multiplier ratio choice**
- ***1/f* can obscure $f_o/2Q$**

  [Rubiola-Giordano 2007]
- **Resonator *1/f* not at VHF**





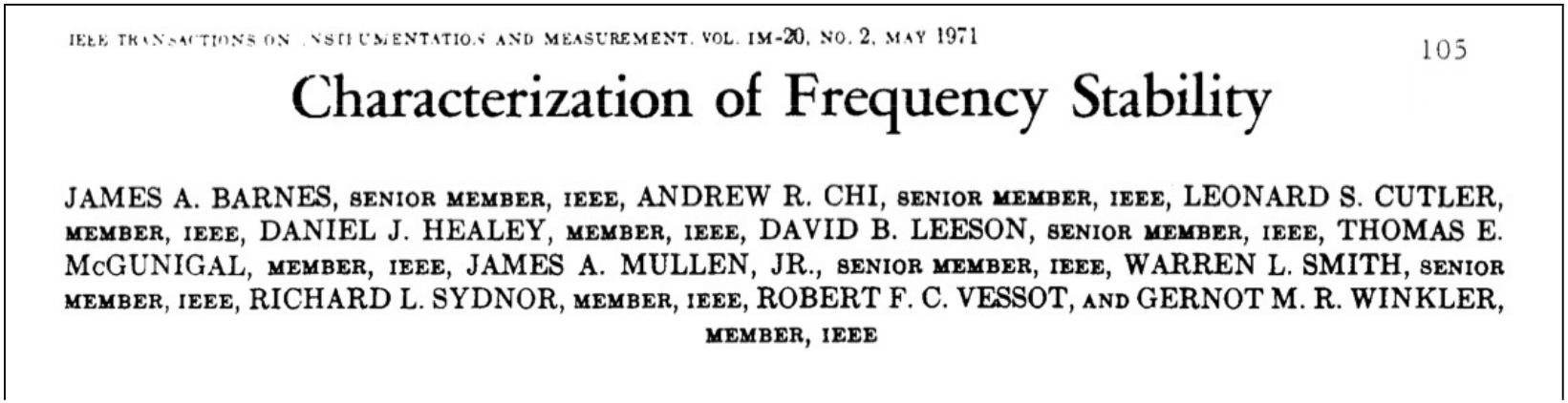
Noise mixed from harmonics



Assumptions: $Q \propto 1/f_o$ & FKT/P_s is constant

1971: Committee as Author

- **Paper as basis of IEEE Std 1139**



IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT, VOL. IM-20, NO. 2, MAY 1971 105

Characterization of Frequency Stability

JAMES A. BARNES, SENIOR MEMBER, IEEE, ANDREW R. CHI, SENIOR MEMBER, IEEE, LEONARD S. CUTLER, MEMBER, IEEE, DANIEL J. HEALEY, MEMBER, IEEE, DAVID B. LEESON, SENIOR MEMBER, IEEE, THOMAS E. McGUNIGAL, MEMBER, IEEE, JAMES A. MULLEN, JR., SENIOR MEMBER, IEEE, WARREN L. SMITH, SENIOR MEMBER, IEEE, RICHARD L. SYDNOR, MEMBER, IEEE, ROBERT F. C. VESSOT, AND GERNOT M. R. WINKLER, MEMBER, IEEE

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Limits and Extensions of Model

- ***F* re nonlinearity, impedances?**
  - **High-Q OK** [Penfield 1966]
  - **Large-signal NF predicted, measured** [Cibiel, et al 2004]
  - **Strongly nonlinear** [Hajimiri-Lee 1998, Demir et al 2000]
- **Delay lines, filters, multiple resonators**
  - **Group delay τ instead of *Q*** [Driscoll 1995]
- **Flicker of resonator — High Q?** [Cutler-Searle 1966, Walls et al 1992]
  - **Added term** [Everard 2000, Rohde 2000, Rubiola 2009]
- **Near-carrier PSD to RF?**
- [Medhurst 1955, Mullen-Middleton 1957, Rutman-Walls 1991, Chorti-Brooks 2006]
- **AM to PM?** [Hearn 1985, Levantino 2002]
  - **Use to reduce PM** [Hati et al 2014]
- **Vibration** [Johnson 1964, Filler 1988, Vig et al 1992,
- Driscoll 2007, Hati et al 2008]

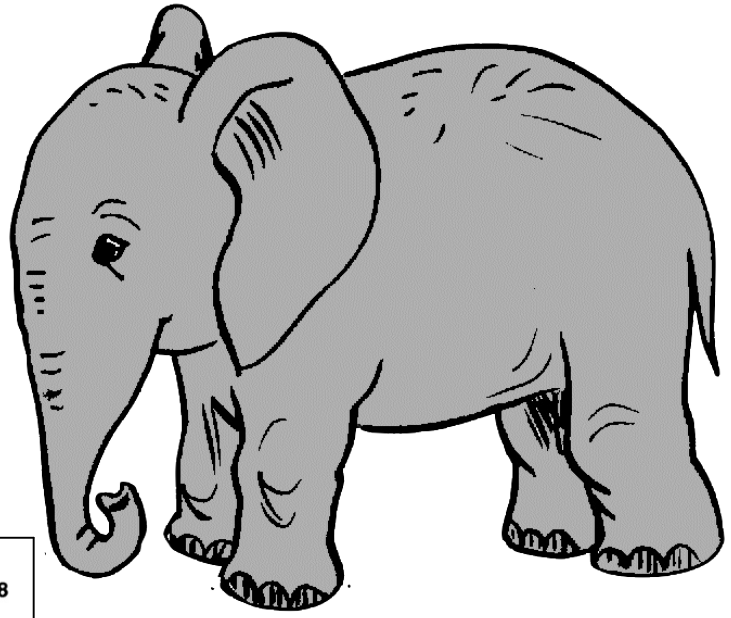
Many excellent papers

Standards and Books

- **IEEE Std 1139 — 1988, 1999, 2008**
- **Everard 2000**
- **Rohde et al 2005**
- **Vendelin, et al 2005**
- **Riley NIST 1065 2008**
- **Rubiola 2009**

1139-1988

IEEE Standard Definitions of
Physical Quantities for Fundamental
Frequency and Time Metrology



Now we can see the whole elephant

Lessons Learned

- **Find mentor & sponsor**
  - **Specialize, don't let them down!**
- **Find unsolved problem in your field**
  - **Not too late in publication cycle**
  - **Read original papers**
- **Learn adjacent "guilds," then synthesize**
  - **RF + semiconductors**
  - **Wireless + networks**
- **Understand standards process**
  - **Publish, join, don't miss out**
  - **Bring into widespread use**