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Heaviside's Disclosure.
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proximity must accommodate itself to them. A part of the wave, however, really does go off into space with some loss of energy at a sharp corner by its own natural tendency to keep going, but the wire serves to guide the disturbance round the corner as a whole, by holding on to the tubes of displacement by their ends. This guidance is obviously a most important property of wires.

There is something similar in "wireless" telegraphy. Sea water, though transparent to light, has quite enough conductivity to make it behave as a conductor for Hertzian waves, and the same is true in a more imperfect manner of the earth. Hence the waves accommodate themselves to the surface of the sea, in the same way as waves follow wires. The irregularities make confusion, no doubt, but the main waves are pulled round by the curvature of the earth, and do not jump off. There is another consideration. There may possibly be a sufficiently conducting layer in the upper air. If so, the waves will, so to speak, catch on to it more or less. Then the guidance will be by the sea on one side and the upper layer on the other. But obstructions, on land especially, may not be conducting enough to make waves go round them fairly. Then waves will go partly through them.

The effects of the resistance of the guides are very complicated in general, and only elementary cases can be considered here. Considering the transmission of plane waves in the ether bounded by parallel cylinders; first imagine the ether to be electrically conducting. Then

$$-\frac{dH}{dx}=kE+c\frac{dE}{dt}, \qquad \text{or} \qquad -\frac{dC}{dx}=KV+S\frac{dV}{dt}, \tag{4}$$

expresses the first circuital law, and

$$-\frac{dE}{dx}=\mu\frac{dH}{dt}, \qquad \text{or} \qquad -\frac{dV}{dx}=L\frac{dC}{dt}, \tag{5}$$

expresses the second. The additional quantities here are k the conductivity, and K the conductance per unit length along the wires. The effect is to waste energy at the rate kE^2 per unit volume, or KV^2 per unit length. It is wasted in heating the medium or matter in it, according to Joule's law. Two other effects occur, viz., attenuation of the wave in transit, and distortion, or change of shape, due to reflection in transit. The attenuation cannot be prevented, but the distortion can. For let the medium be conducting magnetically, so that gH^2 is the waste per unit volume, and RC^2 the corresponding waste per unit length. This g is analogous to k, and R to K. Then instead of (5) above, we shall have

$$-\frac{dE}{dx}=gH+\mu\frac{dH}{dt} \qquad \text{or} \qquad -\frac{dV}{dx}=RC+L\frac{dC}{dt}. \tag{6}$$

These are of the same form as (4). So, if there be no electric conductance, but only the new magnetic conductance, the wave of H will be distorted in the same way as that of E was before, and the wave of E in the same way as that of H was before, and there will be attenuation similarly. But if the two conductivities coexist, though the attenuations are additive the distortions are combative, and may therefore destroy one another. This occurs when $R/L=K/S$, or $g/\mu=k/c$. The solution expressing the transmission of a plane wave is now

$$E=\mu vH=\epsilon^{-gt/\mu}f(t-x/v), \tag{7}$$

or

$$V=LvC=\epsilon^{-Rt/L}\,F(t-x/v). \tag{8}$$

The meaning is that signals are transmitted absolutely without distortion, every slab independently of the rest, but with attenuation in transit according to the time factor $\epsilon^{-Rt/L}$.

This would be very curious, even if it could only be imagined to be done by means of the imaginary magnetic conductivity. What is even more remarkable, however, is that it can be closely imitated by means of the real electric resistance, not of the medium outside the wires, but of the wires themselves. Abolish g altogether, but keep in R, which is to mean the steady resistance of the conducting guides per unit length, previously taken as zero. Then

$$-\frac{dC}{dx}=KV+S\frac{dV}{dt}, \qquad \text{and} \qquad -\frac{dV}{dx}=RC+L\frac{dC}{dt} \tag{9}$$

are still the proper equations in certain circumstances, to be mentioned later. Thus equation (8) is still the result, *i.e.*, distortionless propagation along wires. The waste RC^2 is now in the wires, instead of outside. It equals the other waste KV^2. It follows that any ordinary telegraph circuit may be made approximately distortionless by adding a certain amount of leakance, or leakage conductance; for it has L, S, and R already, and a little K. Increase K until $K/S=R/L$. Then the distortion, which may be excessive at first, will gradually disappear, and the signals will be restored to their proper shape, but at the expense of increased attenuation. If K be increased further, distortion will come on again, of the other kind; for at first R was in excess, but now K is.

For example, if R is 1 ohm per kilom., and $Lv=600$ ohms, then

$$\frac{1}{600}=\frac{R}{Lv}=\frac{K}{Sv}=KLv, \text{ so } K^{-1}=360{,}000 \text{ ohms per kilom.}$$

This is the insulation resistance required. Also, the attenuation in the distance x is $\epsilon^{-Rx/Lv}$ or $\epsilon^{-x/600}$; that is, from 1 to ϵ^{-1} in 600 kilom. to ϵ^{-2} in 1200 kilom., and so on.

To understand the reason of the disappearance of the distortion: concentrate the resistance of the wire in detached lumps, with no resistance between them. Let each resistance be r. Similarly concentrate the leakance, each leak being k. Then, between the r's and k's there is natural unattenuated distortionless propagation; so we have only to examine what happens to a slab wave in passing by one of the r's, or one of the k's, to see the likeness and difference of their effects on V and C. First let a positive wave be passing r. Let V_1, V_2, V_3 be corresponding elements in the incident, reflected, and transmitted waves. Then the conditions are

$$V_1+V_2=V_3+rC_3, \qquad C_1+C_2=C_3, \tag{10}$$

and, since $V_1=LvC_1$, $V_2=-LvC_2$, $V_3=LvC_3$, we have the results

$$\frac{V_3}{V_1}=\frac{1}{1+r/2Lv}, \qquad V_1=V_2+V_3. \tag{11}$$

The second of these equations shows that the electrification (and displacement) is conserved. The first shows the ratio of the transmitted to the incident wave. The incident element, on arriving at r, divides into two, both of the same sign as regards V; one V_3 goes forwards, the other V_2 backwards, increasing the electrification behind. Now suppose a slab wave passes by n resistances in succession in the distance x, such that $nr=Rx$, then the attenuation produced in the distance x is the n^{th} power of V_3/V_1, and in the limit, when n is made ∞, it becomes $\epsilon^{-Rx/2Lv}$. This is when there is no leakage.

Next consider the effect of a single leak. The conditions are

$$C_1+C_2=C_3+kV_3, \qquad V_1+V_2=V_3, \tag{12}$$

and the results are

$$\frac{V_3}{V_1}=\frac{1}{1+k/2Sv}, \qquad C_1=C_2+C_3. \tag{13}$$

Here the second result shows that the induction is conserved, instead of the displacement. A part of C_1 is thrown back and increases C behind the leak. The complete attenuation in the distance x, by similar reasoning to the above, is $\epsilon^{-Kx/2Sv}$, when there is leakage without resistance in the wires.

To compare the two cases; in the first, part of V_1 is reflected positively and part of C_1 negatively, in passing a resistance; and in the second, part of V_1 is reflected negatively and part of C_1 positively. The effects are opposite. So if the resistance and the leak coexist, there is partial cancellation of the reflection. This compensation becomes perfect when r and k are infinitely small and in the proper ratio. Then there is no reflection, though increased attenuation. So with R and K uniformly distributed there is no reflection in transit anywhere, provided $R/L=K/S$. The attenuation in the distance x is now $\epsilon^{-Rx/Lv}$.

If the circuit stops anywhere, what happens to the wave depends upon the electrical conditions at the terminus. If it is a short circuit, then the resultant $V=0$ is imposed. This causes complete positive reflection of incident V, and negative of C. If insulated, then the resultant $C=0$, and there is positive reflection of incident C and negative of V. A remarkable case is that of a terminal resistance, say R_1. Then

$$V_1=LvC_1, \qquad V_2=-LvC_2, \qquad V_1+V_2=R_1(C_1+C_2) \tag{14}$$

are the conditions. So the reflected wave is given by

$$\frac{V_2}{V_1}=\frac{R_1-Lv}{R_1+Lv}. \tag{15}$$

V is reflected positively when $R_1>Lv$, and negatively when $<Lv$. If $R_1=Lv$, there is no reflection. The energy of the wave is absorbed by the resistance. So we attain not only perfect transmission, but also perfect reception of signals.

The above method of treating resistance and leakance by isolated resistances and leaks may be applied to find out what in general happens when there is either no leakage or else no resistance in a continuous circuit. Suppose, for example, there is no leakage. Given a charge anywhere, initially without current, how will it behave? If there were no resistance, it would immediately split into two; one with positive magnetic force would move to the right, the other with negative magnetic force would move to the left, both at speed v. Now this is also exactly what happens in a resisting circuit at the first mmoent, namely, the generation of a

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W. H. Dawes.

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