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Reprints from the Early Days of Information Sciences

On the Contributions of Akira Nakashima to Switching Theory

Reprints from History of Information Sciences

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Детаљи из историје информационих наука

Varhaisia tietotekniikan julkaisuja

Перепечатка из истории информационныих наук

情報科学における歴史的論文の復刻

ՎԵՐԱՀՐԱՏԱՐԱԿՈՒՄ ՊԱՏՄՈՒԹՅՈՒՆԻՑ

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Radomir S. Stanković & Jaakko Astola (eds.)

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On the Contributions of Akira Nakashima to Switching Theory

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Switching Theory

2008

Editors' Notice

Most of the material presented in this publication has been generously provided by Prof. Akihiko Yamada, the Principal of Computer Systems and Media Laboratory, Naganuma-cho, Hachioji, Tokyo, Japan, a Member of IPSJ History Committee, who also helped in clarification of many details regarding the publication of reprinted papers. Prof. Yamada prepared related comments that are included at the appropriate places in the text.

This publication has been written and edited by
Radomir S. Stanković and Jaakko T. Astola.

Contents

Reprints from History of Computing Science and Signal Processing

Historical studies about a scientific discipline is a sign of its maturity. When properly understood and carried out, this kind of studies are more than enumeration of facts or giving credit to particular important researchers. It is more discovering and tracing the way of thinking that have lead to important discoveries. In this respect, it is interesting and also important to recall publications where for the first time some important concepts, theories, methods, and algorithms have been introduced.

In every branch of science there are some important results published in national or local journals or other publications that have been no broadly distributed for different reasons, due to which they often remain unknown to the research community and therefore are rarely referenced. Sometimes, importance of such discoveries is overlooked or underestimated even by the inventors themselves. Such inventions are often re-discovered long after, but their initial sources remain almost forgotten, and mostly remain sporadically recalled and mentioned within quite limited circles of experts. This is especially often the case with publications in other languages than the English language which presently dominates the scientific world.

This series of publications is aimed at reprinting and, when appropriate, also translating some less known or almost forgotten, but important publications, where some concepts, methods or algorithms have been discussed for the first time or introduced independently on other related works.

R.S. Stanković, J.T. Astola

On the Contributions

by

Akira Nakashima

to

Switching Theory

Acknowledgments

The Editors express sincere thanks to Prof. Akihiko Yamada of Computer Systems and Media Laboratory, Tokyo, Japan, for useful comments and interpretation of the related literature, as well as for the advices and help in providing copyright clearances for the papers by A. Nakashma and A. Nakashima and M. Hanzawa. Without his help in that matter the realization of this publication will hardly be possible. We also thank Prof. Yamada for the copies of five papers by A. Nakashima and M. Hanzawa reprinted in this issue of the Reprints. Especially many thanks are due to Prof. Yamada for most of other items included in this issue as illustrations as well as for corrections in references and for providing related explanations and comments that are also included in this publications at appropriate places.

The Editors are grateful to Prof. Dr. Tsutomu Sasao of Kyushu Institute of Technology, Iizuka, Fukuoka, Japan, for providing the paper by Prof. Yamada (reference [22]) and introducing us to him.

The Editors are grateful to Mrs. Marju Taavetti, the Librarian of the Library of Tempere University of Technology, and Mrs. Pirkko Ruotsalainen, the Development Manager of the Department of Signal Processing, Tampere University of Technology, for the help in collecting the related literature. For advices in organization of the work in providing copyright clearance, the Editors are grateful to Mrs. Arja-Ritta Haarala, the Director of the Library of Tampere University of Technology. For the help in providing copyright for reprinting various items included in this issue of Reprints, thanks are due to

Kenji Kogure, the Executive Director of IEICE Headquarter Office, Japan, *Kazuko Hayashi*, IEICE Membership Section, *Yukiko Yamamoto* and *Toshio Shimada*, IEEJ Business Promotion Department, *Hajime Nakajima*, General Affairs Department Manager, Yokogawa Electric Corporation, Tokyo, Japan, *Nobuyoshi Nomizu*, NEC, Tokyo, Japan, *Richard A. Shore*, Publisher, Association for Symbolic Logic, *Berendina van Straalen*, Head of Rights, Heidelberg, Germany, *Dr. Dietrich Merkle*, Springer, Heidelberg, Germany, *Hans van Sintmaartensdijk*, Springer, Dordrecht, The Netherlands, *Estella Jap A Joe*, Springer, Dordrecht, The Netherlands, *Angela Fössl*, Springer, Wien, Austria, *Duncan James*, Wiley, Chichester, United Kingdom, *Bradley Johnson*, Wiley, P&T, Hoboken, New Jersey, USA.

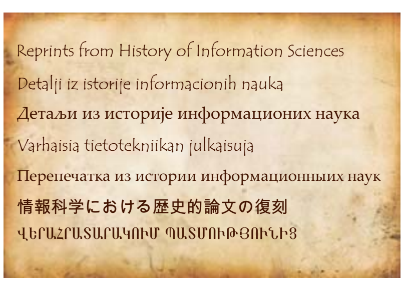
On the Contributions by Akira Nakashima to Switching Theory

Abstract

The present issue of Reprints from the Early Days of Information Sciences discusses research work of Akira Nakashima and his associate Masao Hanzawa on Switching Theory. It presents translations in English of six papers by A. Nakashima and three papers coauthored with Masao Hanzawa and highlights the impact of this work to the research at the time in this area.

Notice

Three kinds of spelling, Nakashima, Nakasima and Nakajima appear in his English papers. Two kind of Chinese characters, 中嶋 and 中島 appear in his Japanese publications.



Radomir S. Stanković, Jaakko Astola

1 Switching Theory

Switching theory and logic design, viewed as practical applications of it, provide mathematical foundations and tools for digital system design that is an essential part in the research and development in almost all areas of modern technology. The vast complexity of modern digital systems implies that they can only be handled by computer aided design tools that are built on sophisticated mathematical models. Development and exploiting in their entire power of such methods is possible if based upon their deep understanding and proper interpretations. In this respect, study of roots and origins of these basic concepts, as well as the way of their introducing to solve particular problems, is of an essential importance.

The Boolean algebra occupies a central role in switching theory, and was a vehicle to transfer circuit design from art and experience into a scientific discipline. Therefore, related basic concepts will be briefly presented in the following section.

1.1 Boolean algebra

Algebra of logic, also called *symbolic logic*, is a method to express logic in a mathematical context. Instead of deling with numeric quantities as in ordinary algebra, it is used to represent the truth value of statement by assigning logic symbols 0 and 1 to two possible truth values *false* and *true*.

It has been derived by George J. Boole in order to permit an algebraic manipulation of logic statements and, therefore, is often called the *Boolean algebra*. It is useful in study of information theory, set theory, probability theory, and represents the basis of switching theory, which is the aspect that will be primarily discussed in this paper.

The Boolean algebra can be defined as follows.

Definition 1 *Consider a set B of at least two distinct elements* 0 *and* 1*. Assume that there are defined two binary operations $\vee$ and $\cdot$ and the unary operation $-$ on B, usually called logic disjunction (OR), conjunction (AND) and negation (NOT).*

An algebraic system $\langle B, \vee, \cdot, -, 0, 1\rangle$ is a Boolean algebra if for any $a, b, c \in B$ the following axioms are satisfied

1.	*Idempotence*	$a \vee a = a,\ a \cdot a = a,$
2.	*Comutativity*	$a \vee b = b \vee a,\ a \cdot b = b \cdot a,$
3.	*Associativity*	$a \vee (b \vee c) = (a \vee b) \vee c,$
		$a \cdot (b \cdot c) = (a \cdot b) \cdot c,$
4.	*Absorption*	$a \vee (a \cdot b) = a,\ a \cdot (a \vee b) = a,$
5.	*Distributivity*	$a \vee (b \cdot c) = (a \vee b) \cdot (a \vee c),$
		$a \cdot (b \vee c) = (a \cdot b) \vee (a \cdot c),$
6.	*Involutivity*	$\overline{\overline{a}} = a,$
7.	*Complement*	$a \vee \overline{a} = 1,\ a \cdot \overline{a} = 0,$
8.	*Identity*	$a \vee 0 = a,\ a \cdot 1 = a,$
9.		$a \vee 1 = 1,\ a \cdot 0 = 0,$
10.	*De Morgan Laws*	$\overline{(a \vee b)} = \overline{a} \cdot \overline{b},\ \overline{a \cdot b} = \overline{a} \vee \overline{b}.$

The postulates 2, 5, 7, and 8 are called the *Huntington postulates* [5], and are sufficient to specify a Boolean algebra, since the remaining six postulates can be derived from them.

Some of these and related results in Boolean algebra have been independently discovered by Akira Nakashima and Masao Hanzawa, as it can be seen in reprinted papers (see, for instance, discussions at pages 49, 53, 104, 105, 107, and elsewhere else).

2 Work of Akira Nakashima in Theory of Logic Networks Design

Akira Nakashima graduated at the Tokyo University, and worked as an engineer at *Nippon Electric Company* (NEC) (Nippon Denki Kabushiki Gaisha) among other task also on the design of relay networks for various purposes.

Nakashima first did an extensive analysis of many case studies of relay networks trying to formulate a unified design theory for such networks. He considered impedances of relay contacts as two-valued variables, and used logic OR and AND operations to represent their series and parallel connections, respectively. Due to that, he formulated a related theory of relay networks by introducing and exploiting some algebraic relations that are a basis of switching theory. For instance, he defined the rules that are nowadays called De Morgan duality expressions (see, for instance, page 59). These results, Nakashima presented without using a symbolic notation in a series of articles in the monthly journal of Nippon Electric Company (NEC) (Nichiden Geppo, in English Nippon Electric - NEC Technical Journal) (see pages 30 to 43) entitled *Theory and Practice of Relay Engineering* [9]. The Telegraph and Telephone Society of Japan engaged Nakashima to give an invited talk at the annual meeting of this society early in 1935. This three hours long talk has been afterward published in [6], [7] (pages 46-57).

In 1936, Nakashima was transferred to transmission engineering, however, being advised to continue this research by Yasujiro Niwa, the Chief Engineer of NEC at that time, and by Yasujiro Shimazu (see page 106), he continued the work after regular office working time with the help of Masao Hanzawa, who remained in the exchange engineering team. This research work by Nakashima has been interrupted when, at the beginning of the Second World War, Nakashima has been again transferred this time to work on radar and wireless communication engineering.

In a joint work with Masao Hanzawa, the theory of Nakashima was elaborated by using also symbolic representations and finally evolved into an algebraic structure, for which Nakashima concluded in August 1938 that it is actually equal to the Boolean algebra.

Notice that papers by Nakashima and also these with Hanzawa, have first been published in Japanese in *The Journal of the Institute of Telephone and Telegraph Engineers of Japan* and *The Journal of the Institute of Electrical Communication Engineers* [1], and then latter translated in a reduced from

[1] The *Institute of Telephone and Telegraph Engineers of Japan* was established in May

and published in *Nippon Electrical Communication Engineering* (page 81).

In [9], which has been published in Japanese in August 1937, the algebra introduced by Nakashima and elaborated in cooperation with Hanzawa was reduced to an algebra of sets by assigning to each partial path a set of times at which its impedance is infinite. In that way, the author was able to state that "theorems and expressions developed in the theory of sets may, therefore, be applied to acting impedance problems of simple partial paths", see the corresponding remark in [1].

For instance, in [10] it is noticed the following correspondence between the algebra of logic and circuits. If A and B are two-terminal circuits, which are called *simple partial paths* by the authors, then $A+B$ and $A{\cdot}B$ correspond to their serial and parallel connections, respectively. The equation $A = B$ states that acting functions of A and B are equal, meaning that A is open (closed) when B is open (closed). Similarly, $\overline{A}$ denotes a simple partial path that is closed when A is open and vice versa. Two simple partial pats that are always closed or open are denoted by p and s, respectively. In the terminology used by the authors, such path have and infinite and zero impedance, respectively.

With this notation (see Fig. 1), that is quite similar to that by Shannon and other authors, Nakashima and Hanzawa defined an algebra which as they realized in 1938 is identical to the Boolean algebra.

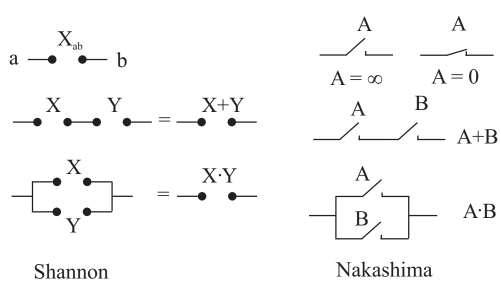


Figure 1: Notation and symbols used by Shannon and Nakashima.

1917. In January 1937 the institute name was changed to the *Institute of Electrical Communication Engineers (IECE) of Japan.* In May 1967 it was renamed again as the *Institute of Electronics and Communication Engineers (IECE) of Japan.* In January 1987 it was reorganized to have four societies and changed the name to the *Institute of Electronics, Information and Communication Engineers (IEICE) of Japan.*

In particular, the authors noticed that their expansion theorem is actually the same what the Boole calls the *law of development*.

In [11], that has been published in Japanese in August 1937, the author is pointing out the links between his algebra and the algebra of sets, which therefore can be used in discussing problems related to simple partial paths and impedances in them [2].

In [12], the authors for the first time refer explicitly to Boole and Schröder.

At page 169, we show the first page of a paper by Shigetoshi Okada arising some questions and providing some comments related to the results by A. Nakashima and M. Hanzawa published in Japanese in 1940. (See reprinted papers 8 and 9). This paper was published in *Journal of the Institute of Electrical Communication Engineers of Japan* in November 1940. At page 177, we show the first page of the paper presenting answers provided by A. Nakashima and M. Hanzawa in the same issue.

In the contemporary research at the time, references to the work by Nakashima are given by C.E. Shannon [17], H. Piesch [14], and O. Pleshl and A. Dushek [15]. At pages 181 to 186 we show the first pages of related papers, as well as the pages where the references to the work by Nakashima and Nakashima and Hanzawa are given.

At pages 187 and 188 we present the review of the work by Nakashima and Nakashima and Hanzawa written by Alonso Church, while his review of the work by Hannsi Piesch is presented at page 189.

Recently, the contributions to the switching theory by Nakashima and Nakashima and Hanzawa, are highlighted and favorably reported in few publications. For example, in [19], and [20], the work by Nakashima [8] has been estimated as the first systematic study of logic circuits. It is especially emphasized the expansion theorem of impedance function [13] of relay circuit and the design theory of two-terminal relay circuits in 1940.

In [4], the work by Nakashima and Hanzawa was favorably reviewed and summarized. (See pages 190-193).

In [21], it is explicitly stated that Akira Nakashima published the first paper on switching theory in the World referring to his paper in 1935. A detailed analysis of the work by A. Nakashima is presented in [22].

In Japan, the work by Nakashima has been continued by Kan-ichi Ohashi, Mochinori Goto [3], Yasuo Komamiya, T. Kojima, and later by many others, see for example, [16], chapter entitled *Switching Theory in Japan*. The related discussion including the references is reprinted at pages 194-199.

References

[1] Church, A., "Review of *Formal logic and switching circuits* by T.A. Kalin, in Proceedings of the 1952 ACM national meeting (Pittsburgh) 1952, Pittsburgh, Pennsylvania May 02 - 02, 1952, 251-257.", *The Journal of Symbolic Logic*, Vol. 18, No. 4, 1953, 345-346.

[2] Church, A., "Review of Nippon Electrical Communicaiton Engineering, by Akira Nakasima and Masao Hanzawa", *The Journal of Symbolic Logic*, Vol. 18, No. 4, 1953, 346.

[3] Goto, M., "Applications of logical equations to the theory of relay contact networks", *Journal of the Institute of Electrical Engineers of Japan*, Vol. 69, No. 726, April 1949, 125-130.

[4] Hanzawa, M., "Origin and Development of Switching Circuit Theory", Chapter 2 in *Switching Circuit Theory*, Nippon Electric Company (NEC), July 17, 1989.

[5] Huntington, E.V., "Sets of independent postulates for the algebra of logic", *Transactions of the American Mathematical Society*, Vol. 5, No. 3, July 1904, 288-309.

[6] Nakashima, A., "Synthesis theory of relay networks", *Journal of the Institute of Telegraph and Telephone Engineers of Japan*, September, 1935.

[7] Nakashima, A., "A realization theory for relay circuits", *Journal of the Instittute of Electrical Communication Engineers of Japan*, No. 150, September 1935, 197-226.

[8] Nakashima, A., "Some properties of the group of single partial paths in the relay circuit", *Journal of of the Institute Telephone and Telegraph Engineers of Japan*, February 1936, 88-95, English translation in *Nippon Electrical Communication Engineering*, March 1937, 70-71.

[9] Nakashima, A., "Algebraic expressions relative to simple partial paths in the relay circuits", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 137, August 1937, in Japanese, Condensed English translation of about half the length of the original paper in *Nippon Electrical Communication Engineering*, No. 12, September 1938,

310-314. Section V, "Solutions of acting impedance equations of simple partial paths".

[10] Nakashima, A., "The theory of two-point impedance of passive networks in the relay circuit (Part 1)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 177, December 1937, Reduced version of part 1 and part 2 (bellow) appears in *Nippon Electrical Communication Engineering*, No. 13, November 1938, 405-412.

[11] Nakashima, A., "The theory of two-point impedance of passive networks in the relay circuit (Part 2)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 178, January 1938, Reduced version of part 1 (above) and 2 *Nippon Electrical Communication Engineering*, No. 13, November 1938, 405-412.

[12] Nakashima, A., Hanzawa, M., "Expansion theorem and design of two-terminal relay networks (Part 1)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 206, May 1940, Condensed English version in *Nippon Electrical Communication Engineering*, No. 24, April 1941, 203-210.

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[18] Stanković, R.S., Astola, J.T., Karpovsky. M.G., "Some historical remarks on switching theory", in Bregović, R., Gotchev, A., (eds.), *Proc. The 2007 International TICSP Workshop on Spectral Methods and Multirate Signal Processing, SMMSP2007*, CD publication, ISBN 978-952-15-1809-6, paper cr1032.

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[20] Watanabe, H., Shinoda, S., "Soul of circuit theory - A review of research activities of graphs and circuits in Japan", *IEEE Trans. Circuit and Systems - Fundamental Theory and Applications*, Vol. 45, No. 1, 1999, 83-94.

[21] Yamada, A., "History of research on switching theory in Japan", *C*, Vol. 123, No. 1, 2003, 1.

[22] Yamada, A., "History of research on switching theory in Japan", *IEEJ Trans. FM*, Vol. 124, No. 8, 2004, 720-726, (in Japanese).

Publications by Akira Nakashima

The following list of publications by Akira Nakashima alone and in a fruitful cooperation with Masao Hanzawa in Switching theory has been compiled and prepared by Prof. Akihiko Yamada [22] [2] who also provided the related comments especially for this issue of the *Reprints*. Some of the papers were reviewed in [2] by Alonso Church or mentioned in his reviews of the work by other authors in the field (see pages 187-189), and also have been reported in the review book by the staff of the Nippon Electric Company (NEC) [4] (see pages 190-193). See also [18].

1. Nakashima, A., "The theory of relay circuit engineering", in the journal *Nichiden Geppo (Nippon Electric)* (see page 29 for the cover page of an issue of this journal) published by *Nippon Electrical Company (NEC)*, November 1934-September 1935, (in Japanese). The current name of the journal is *NEC Technical Journal*. No English version of this paper.

2. Nakashima, A., "Synthesis theory of relay networks", *Journal of the Institute of Telegraph and Telephone Engineers of Japan*, No. 150, September 1935. Title translated also as "The theory of relay circuit composition" and "A realization theory for relay circuits", English version in *Nippon Electrical Communication Engineering*, No. 3, May 1936, 197-226.

3. Nakashima, A., "Reziprozitaetsgesetze", in the journal *Nichiden Geppo - Nippon Electric* published by *Nippon Electrical Company (NEC)*, January 1936 (in Japanese). The current name of the journal is *NEC Technical Journal*. No English version of this paper.

4. Nakashima, A., "Some properties of the group of simple partial paths in the relay circuit", *Journal of the Institute of Telephone and Telegraph Engineers of Japan*, January, February, and March 1936, 88-95, English translation in *Nippon Electrical Communication Engineering*, March 1937, 70-71.

5. Nakashima, A., Hanzawa, M., "The theory of equivalent transformation of simple partial paths in the relay circuit (Part 1), *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 165, October 28,

[2] References at pages 18-20.

1936, published December 1936. Condensed English version of parts 1 and 2 in *Nippon Electrical Communication Engineering*, No. 9, February 1938, 32-39. Reference in Piesch [14] (see page 183) and [15] (see page 185).

6. Nakashima, A., Hanzawa, M., "The theory of equivalent transformation of simple partial paths in the relay circuit (Part 2), *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 167, December 14, 1936, published in February 1937. Condensed English version of parts 1 and 2 in *Nippon Electrical Communication Engineering*, No. 9, February 1938, 32-39.

7. Nakashima, A., "The theory of four-terminal passive networks in relay circuit", *Journal of the Institute of Electrical Communication Engineers of Japan*, April 1937, English summary in *Nippon Electrical Communication Engineering*, No. 10, April 1938, 178-179.

8. Nakashima, A., "Algebraic expressions relative to simple partial paths in the relay circuits", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 173, August 1937, in Japanese. Condensed English translation of about half the length of the original paper in *Nippon Electrical Communication Engineering*, No. 12, September 1938, 310-314. Section V, "Solutions of acting impedance equations of simple partial paths".

9. Nakashima, A., "The theory of two-point impedance of passive networks in the relay circuit (Part 1)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 177, December 1937. Reduced version of part 1 and part 2 (bellow) appears in *Nippon Electrical Communication Engineering*, No. 13, November 1938, 405-412.

10. Nakashima, A., "The theory of two-point impedance of passive networks in the relay circuit (Part 2)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 178, January 1938. Reduced version of part 1 (above) and 2 in *Nippon Electrical Communication Engineering*, No. 13, November 1938, 405-412.

11. Nakashima, A., "The transfer impedance of four-terminal passive networks in the relay circuit", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 179, February 1938. Condensed English version in *Nippon Electrical Communication Engineering*, No. 14, December 1938, 459-466.

12. Nakashima, A., Hanzawa, M., "Expansion theorem and design of two-terminal relay networks (Part 1)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 206, May 1940. Condensed English version in *Nippon Electrical Communication Engineering*, No. 24, April 1941, 203-210.

13. Nakashima, A., Hanzawa, M., "Expansion theorem and design of two-terminal relay networks (Part 2)", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 209, August 1940. Condensed English version in *Nippon Electrical Communication Engineering*, No. 26, October 1941, 53-57.

14. Nakashima, A., "Theory of relay circuit", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 220, March 1941, 9-12. No English translation of this paper.

> This is a kind of short tutorial. The first and the second page of this tutorial are shown at pages 57 and 59. This page contains basic postulates and theorems in the Boolean algebra. The first reference in this paper is in Japanese, and other three are
>
> 1. Boole, G., *An Investigation of the Laws of Thought*, London, 1854.
> 2. Schröder, E., *Vorlesungen über die Algebra der Logik*, 1890.
> 3. Couturat, L., *The Algebra of Logic*, 1914.

15. Nakashima, A., "Theory of relay circuit", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 220, July 1941, 397-406. No English translation of this paper.

> This is the speech delivered by Akira Nakashima at the general assembly of the IECEJ on 26th April 1941. It covers his major research results. The first and the third page of this paper are reprinted at pages 65 and 67. At the third page of this paper we see a table with basic postulates and theorems in the Boolean algebra. Besides references as in the item 14, Nakashima has mentioned the *Journal of Symbolic Logic*, 1936.

In item 12, there are references to the work by Boole and Schröder mentioned above. In item 13, there is a reference to the work
Bernstein, B.A., "Postulate for Boolean algebra involving the operation of complete disjunction", *Annals of Mathematics*, Vol. 37, No. 2, April 1936, 317-325.

In 1940, Nakashima published the following paper
Nakashima, A., "Current status of wired communication technologies in the United States", *Journal of the Institute of Electrical Communication Engineers of Japan*, August, 1940, 477-484 (in Japanese).

> The paper is the transcription of the speech of Akira Nakashima at the *IECE* lecture meeting on June 19, 1940, which is a trip report to the US. The contents is about wired communication technologies in general. (See pages 58 and 59).

Maybe the last publication of Nakashima is the following:
Nakashima, A., "Reminiscences of the Switching Network Theory", *Journal of the Institute of Electronics and Communication Engineers of Japan*, December 1970, 1658-1661.

> This is a history of Nakashima's researching on switching theory. It includes the brief description about his visit to Bell Labs and the meeting with Claude Shannon at *AIEE Winter Convention.* He passed away on the 29th October 1970. Therefore this article was published after his death.

First page of this paper is shown at page 75.

The condolence on Nakashima's death was written by Zen-iti Kiyasu, Kan-ichi Ohhashi and Yasujiro Shimazu (to whom Nakashima wrote an acknowledge in his second paper, see page 120) and was published in the *the Institute of Electronics and Communication Engineers*, Vol. 54, No. 3, March 1971.

First page of the condolence is shown at page 79.

The following table with an analysis of the work by A. Nakashima is reprinted from Yamada, A., "History of research on switching theory in Japan", *IEEJ Trans. FM*, Vol. 124, No. 8, 2004, 720-726, (in Japanese).

表1　中嶋章のスイッチング理論に関する論文一覧

Table 1.　Akira Nakashima's papers on switching theory.

No.	Title	Date	English Version**	Contents	Reference
1	継電器工学の理論と実際 The Theory and Practice of Relay Circuit Engineering	1934-11～ 1935-9　(N)	—	Definitions and theory of relay circuits.	
2	継電器回路の構成理論 The Theory of Relay Circuit Composition	1935-9 講演 1935-9 (T)	1936-5	De Morgan' s theory. Relay circuit Categorization.	
3	反転の理(Reziprozitaetsgesetze)に就いて Reziprozitaetsgesetze	1936-1　(N)	—	Reziprozitaetsgesetze	
4	継電器回路における単部分路群の性質について Some Properties of the Group of Simple Partial Paths	1936-2　(T)	1937-3	Application of group theory.	
5*	継電器回路における単部分路の等価変換の理論 (其の一) The Theory of Evolution Equivalent Transformation of Simple Partial Paths in the Relay Circuit (Part 1)	1936-12 (T)	1938-2	Distribution and elimination law. Series-parallel transformation. Algebraic expression.	H. Piesch [23]
6*	継電器回路に於ける単部分路の等価変換の理論 (其の二) The Theory of Evolution Equivalent Transformation of Simple Partial Paths in the Relay Circuit (Part 2)	1937-2　(T)			
7	継電器回路における負性四端子回路網の理論について The Theory of Four-Terminal Passive Networks	1937-4　(T)	1938-4 (summary)	Four-terminal passive network theory	C. E. Shannon [25]
8	継電器回路に於ける単部分路の関係式論 Algebraic Expressions Relative to Simple Partial Paths in the Relay Circuit	1937-8 (C)	1938-9	Interpretation based on group theory. Principle of dual circuits	C. E. Shannon [25]
9	継電器回路に於ける負性回路網の二点間作動インピーダンス理論 (その一) The Theory of Two-Point Impedance of Passive Networks in the Relay Circuit (Part 1)	1937-12 (C)	1938-11	Theory of two-point impedance of passive networks.	C. E. Shannon [25]
10	継電器回路に於ける負性回路網の二点間作動インピーダンス理論 (その二) The Theory of Two-Point Impedance of Passive Networks in the Relay Circuit (Part 2)	1938-1 (C)			
11	継電器回路における負性四端子回路網の Transfer Impedance について The Transfer Impedance of Four-Terminal Passive Networks in the Relay Circuit	1938-2 (C)	1938-12	Transfer impedance.	C. E. Shannon [25]
12*	継電器回路に於けるインピーダンス関数の展開定理と二端子回路網の設計理論 (其の一) Expansion Theorem and Design of two Terminal Relay Networks (Part 1)	1940-5 (C)	1941-4	Expansion theorem and design of two terminal relay networks.	
13*	継電器回路におけるインピーダンス関数の展開定理と二端子回路網の設計理論(其の二) Expansion Theorem and Design of two Terminal Relay Networks (Part 2)	1940-8 (C)	1941-10		
14	継電器回路の理論 Theory of Relay Circuit	1941-4 講演 1941-7 (C)	—	Theory of relay circuits.	

Note: * Co-authored with Masao Hanzawa,　** 英文掲載：Nippon Electrical Communication Engineering への掲載時期

(N)：日電技報 NEC,　(T)：電信電話学会雑誌 J. of Telegraph and Telephone Soc.,　(C)：電気通信学会誌 J. of IECEJ

The first and the third paper by Nakashima mentioned above were published in *Nippon Electric Technical Journal*, whose formal name in English is **Nichiden Geppo** (see page 30). Page 29 shows the first page of the article by Akira Nakashima in Nichiden Geppo, Vol. 11, No. 11, November 1934. The three pages of Vol. 12, No. 11, January 1935, of this journal are shown at pages 35, 37, 39 and the rear cover page is shown at the page 41. The first page of the paper by Akira Nakashima "Theory and practice of relay circuit engineering (Number five)", is shown at page 44.

This series of publications by Nakashima consists of the following seven papers:

Number 1: Vol. 11, No. 11, pp.1-7 (Nov. 1934)
Number 2: Vol. 12, No. 1, pp.9-23 (Jan. 1935)
Number 3: Vol. 12, No. 2, pp.8-22 (Feb. 1935)
Number 4: Vol. 12, No. 3, pp.18-26 (Mar. 1935)
Number 5: Vol. 12, No. 4, pp.1-13 (Apr. 1935)
Number 6: Vol. 12, No. 5, pp.1-12 (May 1935)
Number 7: Vol. 12, No. 9, pp.1-10 (Sep. 1935)

There is another paper by Nakashima in *Nichiden Geppo*:

Nakashima, A., "Transient phenomena during releasing operation of a relay with two parallel coils", *Nichiden Geppo (Nippon Electric)*, Vol. 12, No. 4, April 1935, 14-18.

> This is the addendum to the above Number 3 published in February 1935.

In November 1935, Nakashima published another paper related to the same subject

Nakshima, A. "An approximation solution of the transient phenomena during the operating and releasing action of telephone relays having many varied secondary circuits", *Journal of the Institute of Telephone and Telegraph Engineers of Japan*, November 1935, 889-892, (in Japanese). (See page 49).

Pages 51 and 53 show first pages of the papers reported as the items 5 and 6 in the list of publications of Akira Nakashima in Switching Theory. Pages 59 and 61 show the first and the second page of the item 14.

中嶋 章

Akira Nakashima

Photo from the Archive of Corporate Communication Division (CCD) of
Nippon Electric Company, Ltfd., (NEC)
provided by courtesy of Mr. Nobuyosi Nomizu of NEC and Prof. Akihiko Yamada

The economic depression started in October 1964 in Japan.
In November, in the midst of depression,
Akira Nakajima
was appointed president of *Ando Electric Company.*

Akira Nakashima
President of Ando Electric Company, 1964

Photo from the Archive of Corporate Communication Division (CCD) of
Nippon Electric Company, Ltfd., (NEC)
provided by courtesy of Mr. Nobuyosi Nomizu of NEC and Prof. Akihiko Yamada

The page 74 of the book
Fifty years of Ando Electoric Co.,Ltd
published in September 1983.

" 昭和40年 " in the upper left corner of page 74
of the Ando book shows the year of 40 in Showa era.
It corresponds to the year of 1965.

The title is "Resolution to become a world's leading instrument manufacturer".
The subtitle is "Depression in the year of Showa 40 (1965)"

This page shows the New Year message in January 1965
he told all employees as follows and encouraged them:

"We private company have to earn by ourselves to survive.
We shouldn't rely on others. If we are not cautious,
we will fall and have to disappear.
Be ambitious to become a world's leading instruments manufacturer!"

Nakajima promoted education for managers and sped up
the development of digital instrument systems.
In 1966 they started to develop IC testers.

(Comments written by **Prof. Akihiko Yamada**,
who provided the reprinted page by courtesy of
Mr. Hajime Nakajima, General Affairs Department Manager,
Yokogawa Electric Corporation which incorporates former Ando Electric.)

昭和**40**年

世界一流への決意

◉40年不況

40年不況は，39年10月に始まり，40年10月に底を打った。しかしこれまでの不況とは様相を異にし，30年代の日本経済と40年代の日本経済を区分する重要な画期であった。この真っ最中，前年の11月に就任した中嶋社長は，新年に際し「私企業は自分で働いて自らの口を養うもので，他力依存は不可の覚悟が必要。油断すれば容赦なく没落し消え去らねばならない。安藤電気を世界一流の測定器メーカーにする大志を抱いて欲しい」と全社を激励した。

中嶋　章

当時の安藤電気にとって，世界一流の測定器メーカーをめざすことはやはり大志である。ひとつケタが違う感じであった。中嶋社長は，当社の実体把握に努めた。計測業界は依然として停滞，販売競争も熾烈であり，当社も売り上げは伸び悩んでいた。中嶋社長は，実体把握の段階で，当社の伸び悩みは景気の停滞だけではなく，扱い機種が壁にぶつかり，この点を含めての企業体質の改善が必要と考えた。

◉ＮＨＫより感謝状

ＮＨＫのＦＭ放送運用上不可欠な各種機器（ＦＭステレオ用測定器，監視用受信機並びに中継用受信機）を完成させ，ＦＭ放送の実用化に貢献した。３月29日，ＮＨＫより当社はＦＭ放送網の建設に際し「高精度かつ安定した機器を短納期の要請に応じ完成させた」として感謝状が贈られた。

◉管理者教育に力を入れる

管理者教育の一環として，５月，昭和40年度業績向上のための着眼点として次の項目をかかげた。

①受注拡大と売り上げ増進②市場調査と製品開発③管理者，監督者の教育並びに訓練と士気の昂揚④生産管理の合理化⑤鷲津工場の強化育成⑥発注業務の改善⑦原価低減⑧品質の向上⑨創意工夫の活発化⑩経費節減。

以上の各項目について細かく指示を出し，各部長，工場長に担当項目の具体的な手段方法を答申させ，実施結果について報告を求めた。

◉開発速度を早める

電電公社をはじめとして大手通信機メーカーにおけるＰＣＭ通信用計測器の需要が急増，これに的をしぼって技術力を集中し，開発を急いだ。当時，同軸ケーブル用測定器はより高周波に向かい，公社でも高周波同軸用測定器を要望していたが，当社は水晶制御による周波数ロック・イン機構を持つ高精度の測定器を開発した。

また，電電公社をはじめ大手通信機メーカーにおけるＰＣＭ通信用計測器の需要が急増し，これに対応して，各種の測定装置の開発を完了した。データ伝送測定器の開発品が一部完成した。ディジタル計測器への急速な展開も進んだ。

◉創立者安藤三衛の社葬

11月６日，東京都目黒区碑文谷の円融寺において社葬が行われ，遺族親戚，友人，会社役員をはじめ多数参列した。

The first page of the article by **Akira Nakashima** in
Nichiden Geppo
Vol. 11, No. 11, November 1934.

繼電器回路工學の理論と實際（其の一）

技術課　中　嶋　章

緒　言

近時種々精妙なる自働電話交換方式の發達に伴ひ所謂繼電器回路なるものも次第に複雜に且つ巧緻を極めて來た。而して其の思想及び考案は他の種々の工學上の分野例へば强電方面の發變電所の自働的管理及び夫等の遠方監視制御、一般の警報等の表示、弱電方面の傳送回路の種々なる自働的制御等頗る廣範圍に活用されて、其の巧味を縱横に發揮して居る。
又眞空管回路の發達につれて之と組合す事に依り從來普通の電磁繼電器では到底得られなかつた巧妙な繼電器回路も生れて來た。

斯くの如く繼電器回路の發達と其の優れた自働性に依り其の適用される分野は益々擴まるべく、十分なる理解の下になされる應用に依て種々なる工學上の要求を十分滿足せしめ得るであらう。

玆に所謂繼電器回路なるものに就て少しく考察して見たいと思ふ。從來此の方面の考へ方に就て未だ何等纒つたものの發表を見ない。其の內容性質の如何を問はず從來得た智識を整理し一つの體系を與へておく事は旣得の智識を明確に把握する爲のみならず其の一系の學問の將來性に對する洞察力を養成し更に進んで其の發展を促進する點に於て大いに意義あるものと思はれる。繼電器回路に就ては特に之を痛感してゐる。敢て「繼電器回路工學」と潛稱して筆をとる所以、從て以下述べる所も獨斷的に過ぎるもの多々あると思ふ。之は最初の試みとして御寬容願ひ度い。又其の考察の方法及び內容も未だ粗笨なるを免れないであらう。大方の御教示と御叱正とを乞ふ次第である。

一般に繼電器回路は或る基本現象と夫に對應する所望の現象との仲介者として最も合理的に要求を滿足させんとする一つの手段である。從て之が應用に際しては基本現象と所望現象とに對する理解の外に尙繼電器回路の機能に就て十分なる智識を必要とする。從來前者の專門家は繼電器回路の構成を其の專門家に委囑するに當り繼電器回路に對する理解少き爲め基本現象及び所望現象の條件の提示に不十分なる憾み多く其の間種々なる損失を免れぬ場合が往々ある。

The cover page of

Nichiden Geppo

Vol. 12, No. 11, December 1, 1934.

The cover of Nippon Electric (NEC Technical Journal). The formal name of the journal in English is "Nichiden Geppo" as shown in it. The original Japanese name is "日電月報".

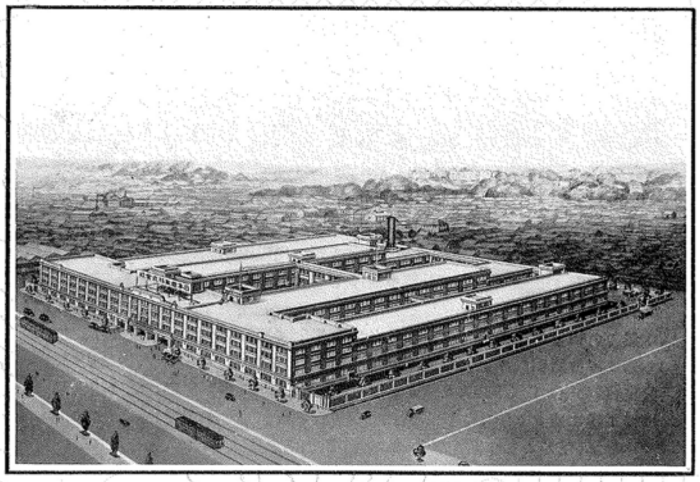
"Nichiden Geppo" is the Romanized (alphabetic) expression of "日電月報". "日電" is the abbreviation of "日本電気 (Nippon Denki)". "Denki" means "electric" in English. "月報" means "a monthly journal". Therefore "Nichiden Geppo (Nippon Electric)" or "Nichiden Geppo (Nippon Electric monthly journal)" would be an appropriate expression in English. Just "Nippon Electric" might also be all right as "Nippon Electric" is shown in the heading of each page.

Notice that the *Institute of Telegraph and Telephone Engineers of Japan* uses the name "Nippon Electrical Communication Engineering" for their journal.

第十二卷

日電月報

第四號

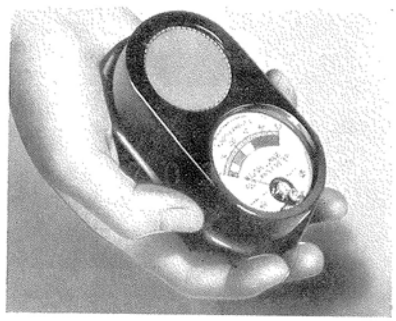


日本電氣株式會社

The second page of
Nichiden Geppo
Vol. 12, No. 11, December 1, 1934
with an advertisement of Weston's illumiometer.

７０３型
ウエストン照度計

容易に而も確實に照明測定の出來る萬人向のポケツト型照度計で、時刻を時計で見るが如き容易さを以て一目にして照度を知る事の出來る便利な照度計であります。



本照度計は、光電池を利用するもので、此の光電池が光の勢力を電氣の勢力に變換して目盛上で正確な照度を示します。

目盛は五つの色の部分に區分せられ、色の部分は更に五十度に分けられ本器の背面にある表により種々な仕事や場所に適する照度を示します。卽ち赤は屋内照明に不適當な照度、黃は階段、倉庫に適する照度、綠は店舗、教室、事務室に向く照度、藍は裁縫室、製圖室、工場に向く照度、紫は精密器械の工場に適する照度といふが如く種々な場所に適する照度を示します。

本器は、電源として電池又は其の他の附屬品を要せず、永久的使用に耐へる經濟的照度計であります。好き照明を以て能率好く仕事をなさんと欲する方々は是非とも利用すべきものであります。

The third page (index page) of
Nichiden Geppo
Vol. 12, No. 4, December 1, 1934.

目　次

The rear cover of
Nichiden Geppo
Vol. 12, No. 11, December 1, 1934.

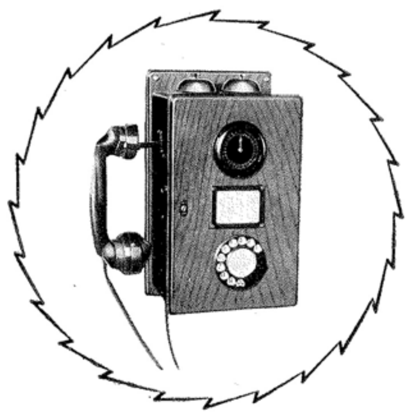
NICHIDEN GEPPO

Published Monthly by the Nippon Electric Company Ltd.
2 Mita-Shikoku-machi, Shiba-ku, Tokyo, Japan.

Vol. 12 December 1st., 1934 No. 11

N.E. 式個別呼出電話

一回線に多数の電話機を接續して、その各電話機が相互間の選擇呼出又は全電話機若しくは任意の一群の電話機の同時呼出を可能とする電話であります。



個別呼出電話用
8483-A 壁掛電話機

N.E. 式個別呼出電話は、働作は確實、設備は簡單、保守は容易であつて、經濟的に施設出來る新方式の電話であります。

用途としては、鐵道の各驛間連絡通信用、電力會社の發電所及變電所等の連絡通信用、警察署、消防署等の連絡通信用、鑛山、造船所、其他工場用の如き種々なる用途が擧げられます。

The first page of the paper by
Akira Nakashima in
Nichiden Geppo

Vol. 12, No. 4, April 1935, 1-13.

The title of the paper is
"継電器回路工学の理論と実際"（其の五)
"Theory and practice of relay circuit engineering (Number five)".
(The English title is not shown in the original Japanese paper.)

繼電器回路工學の理論と實際（其の五）

（第十二卷第三號に續く）

技術課　中嶋　章

第三篇　繼電器の時間的作働特性（續）

本篇の前々號及び前號に於ては繼電器の受入れる制御勢力と之から變換される仲介勢力の時間的作働特性に及ぼす影響に就て論じたが、仲介勢力の量的大さと云ふものは絶對的のものではなく、常に繼電器接點の作働に對して賦課された對抗勢力に對する相對的のものである。又前號に於て數式を以て示した電磁繼電器の作働時間は單に作働開始時間のみを意味するもので、一般に眞の作働時間の一部を表はしてゐるに過ぎない。即ち今眞の作働時間 τ の內容を考へて見ると

$$\tau = t_1 + t_2 + t_3 \cdots\cdots(37)$$

茲に　t_1 ＝ 作働開始時間　　t_2 ＝ 接點の變位時間

t_3 ＝ 接點の"躍り"の時間

と表はされ、先に論じたものは（37）式中の t_1 のみである。t_1 の終りに於て磁束に依る接極子の吸引力は接點彈條の對抗力に打勝つて始めて變位運動を開始するが、其の機構上定まる慣性の爲に直ちには一定速度の運動を起さず、又途中の運動速度は對抗勢力の各變位位置に於ける値の製肘を受け從て所定の作働位置まで變位するには或る時間 t_2 が必要である。普通に τ は此の二者の和であるが、若し接點の躍り (bouncing 或は chattering) が起る時は此の繼續時間 t_3 をも考慮に入れなければならぬ。

以上の如く時間的作働特性を支配する因子としては前號に引續いて尙之等の對抗勢力及び接點に關するものを考へる必要がある。依て以下本號では之等の點を少し考へて見たい。

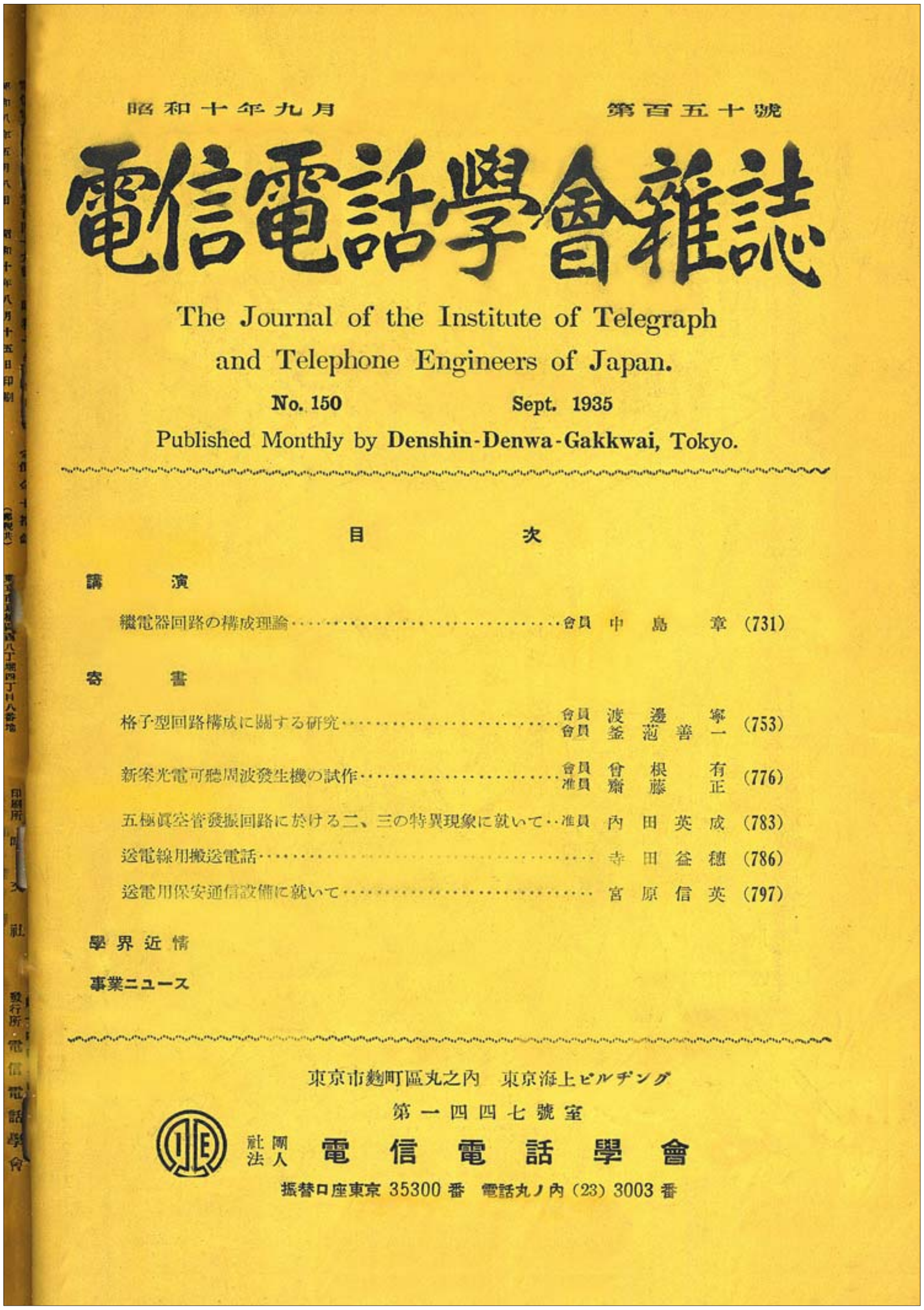
第四章　對抗勢力に關する因子の影響

341　對抗勢力の强さと靜的變化

機械的接點機構を持つ繼電器の對抗勢力に就て、考へて見る。其の種類としては

a. Canti-lever 式彈條の呈する對抗力

The cover page with contents of
Journal of the Institute of Telephone and Telegraph Engineers of Japan
No. 150, September 1935.
The first paper is A. Nakashima's "Synthesis Theory of Relay Networks."



昭和十年九月　第百五十號

電信電話學會雜誌

The Journal of the Institute of Telegraph and Telephone Engineers of Japan.

No. 150　**Sept. 1935**

Published Monthly by **Denshin-Denwa-Gakkwai**, Tokyo.

目　次

講　演

寄　書

學界近情

事業ニュース

東京市麴町區丸之內　東京海上ビルヂング

第一四四七號室

社團法人 電信電話學會

振替口座東京 35300 番　電話丸ノ內 (23) 3003 番

The rear cover

Journal of the Institute of Telephone and Telegraph Engineers of Japan
No. 150, September 1935.


電燈線で動く
ニホン周波電氣時計
發賣元
日本電氣株式會社
東京市芝區三田四國町二
取扱容易・示時正確
堅牢耐久・價格至廉
N.S.C
出張所
大阪支社
大阪市東區北濱四丁目十六番地
代理店

The page 33 (341) of
The Journal of the Institute of Telephone and Telegraph Engineers of Japan
October 1935
a very brief report of Nakashima's lecture meeting in *the Institute Column*

Academic Lecture Meeting

September 19 at Big Hall of Denki-Kurabu in Yuraku-cho, Kojimachi-ku. The lecture meeting (the seventh in 1935) was held. 289 people including 93 members and 113 associate members attended it.

Title: The Theory of Relay Circuit Composition
Speaker: Akira Nakashima, Nippon Electric Company.

測定

ハ ベンチデイン試藥による電蝕檢測

○入 會 下記諸君の入會を承認す。

(1) **會員** (35 名)

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(2) **准員** (116 名)

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九月十九日　麴町區有樂町電氣俱樂部大講堂に於て講演會（本年第七回）開催，下記講演ありたり（出席會員 93 名，准員 113 名，傍聽 83 名，計 289 名）

繼電器回路の構成理論

日本電氣株式會社

會員 中島 章君

第三回工學會大會規則

第一章　會ノ名稱，時期及場所

第一條　本大會ハ之ヲ第三回工學會大會ト稱シ昭和十一年四月東京ニ於テ之ヲ開催ス

第二章　會ノ目的

第二條　本大會ハ東亞地方ニ於ケル各國ノ工學及工業關係者協同シテ次ニ揭クル事項ヲ遂行スルヲ以テ目的トス

一，工學及工業ニ關スル論文ノ發表及意見ノ交換ヲ爲シ以テ智識ヲ增進シ且懇親ヲ圖ルコト

二，發表ノ論文，意見並決議ヲ記錄シテ工學及工業ニ關スル參考資料ト爲スコト

第三章　會議及施設

第三條　本大會ハ總會及部會ノ二種トシ更ニ第二條ノ

The first page of the paper

Nakshima, A. "An approximation solution of the transient phenomena during the operating and releasing action of telephone relays having many varied secondary circuits"

Journal of the Institute of Telephone and Telegraph Engineers of Japan

November 1935, 889-892.

寄　　　　書

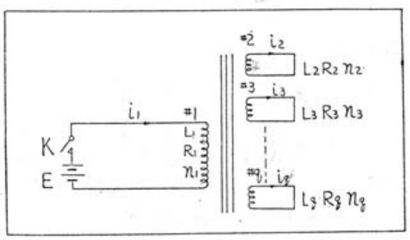
多數の二次電路を有する電話用繼電器の作働過渡現象の近似解に就て

會員　中　島　　章
(日本電氣株式會社)

梗概　多數の二次電路を有する電話用繼電器の各電路間の磁氣結合が完全であると假定して其の働作時及び復舊時の過渡現象の近似解を求め，此の解に基いて働作開始時間及び復舊開始時間の簡單な表示式を導き最後に復舊時に於ける勢力の消散狀態を考察した。

I.　緒　　　言

一般に電話用繼電器としては鐵心，銅環若しくは銅管，或は二次線輪など働作線輪に對する二次電路を多數に有するものが屢々使はれ，其の遲緩作働形態を利用して繼電器回路に重要な役割を演ぜしめる事は周知の事柄である。之が働作或は復舊する際の過渡現象及び作働時間を數式的に求め其の物理的意義を明かにする事は二次電路が單一の場合は簡單ではあるが，一般に二個以上となると困難である。筆者は働作線輪及び各二次電路の總ての相互間の磁氣結合が完全であると假定して，斯かる繼電器の作働過渡現象及び作働時間を近似的に求めて見た。但し過渡現象は接極子が運動し始めるまでに限定する。蓋し若し接極子が運動を行ひ繼電器としての眞の作働の全般を考察するとすれば接極子の運動に關する力學的の問題及び接極子の運動に伴ふ磁路の變化等の爲に複雜な積分方程式の問題となり解決が困難となるからである。從つて玆に取扱ふ作働時間は，第一圖に示す如く一次電路即ち働作線輪回路を閉成或は開放した時から接極子が將に動き始めんとする時まで，即ち働作開始時間及び復舊開始時間のみに限定して論ずる事にする。



第　一　圖

An Approximate Solution of the Transient Phenomena During the Operating and Releasing Action of Telephone Relays Having Many Varied Secondary Circuits. Akira Nakashima, member (Nippon Electric Co. Ltd.)
(J.I.T.T.E. November, 1935)

II.　働作時の過渡現象及び働作開始時間

第一圖に示す如く二次電路は一般に $(q-1)$ 個存在するものとして，開閉器 K を閉ぢた場合の各電路の電流は下記の q 元一次聯立方程式で與へられる。但し玆に p はヘヴイサイドの演算子である。

$$\left.\begin{aligned}
&(L_1p+R_1)i_1+M_{12}pi_2+M_{13}pi_3\\
&\qquad+\cdots\cdot+M_{1q}pi_q=E\\
&M_{21}pi_1+(L_2p+R_2)i_2+M_{23}pi_3\\
&\qquad+\cdots\cdot+M_{2q}pi_q=0\\
&M_{31}pi_1+M_{32}pi_2+(L_3p+R_3)i_3\\
&\qquad+\cdots\cdot+M_{3q}pi_q=0\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&M_{q1}pi_1+M_{q2}pi_2+M_{q3}pi_3\\
&\qquad+\cdots\cdot+(L_qp+R_q)i_q=0
\end{aligned}\right\}\cdots(1)$$

周知の如く (1) 式から各電路の電流を行列式の形で求めると次の樣になる。

$$i_k=(-1)^{k-1}\frac{D\binom{1}{k}}{D}E,$$

$$k=1,2,3,\cdots\cdot,q\cdots\cdots\cdots\cdots(2)$$

玆に D 及び $D\binom{1}{k}$ は夫々下記の行列式を意味し，D は後に示す如く $D\neq 0$ と考へられる故に (2) 式は (1) 式の解として妥當である。尚本稿では方列の横列を行，縱列を列と呼ぶ事に規約する。

The first page of the paper

Nakashima, A., Hanzawa, M.

"The theory of equivalent transformation of
simple partial paths in the relay circuit (Part 1)

Journal of the Institute of Telephone and Telegraph Engineers of Japan

No. 165, October 28, 1936, published December 1936.

$-10405395\,x^{-5}+164189025\,x^{-7}$
$-344594250\,x^{-9})$

$f_9(j\,10)=0.229+j\,1.489$

$F_9(j\,10)=-6.03+j\,4.61$

$f_{11}(j\,10)=1.060-j\,2.289$

$F_{11}(j\,10)=8.76-j\,9.48$

$f_{13}(j\,10)=-4.046+j\,6.379$

$F_{13}(j\,10)=-36.24+j\,54.65$

（昭和11年10月14日）

繼電器回路に於ける單部分路の等價變換の理論

（其　の　一）

會員　中　島　　章　　會員　榛　澤　正　男
（日本電氣株式會社）

內容梗概　先に筆者の一人が發表した「繼電器回路の構成理論」(1) 中に述べた單部分路に關して其の作働機能を變化せしむる事なく接續形式を變換する場合卽ち等價變換を施行する際の變換理論を考察して，若干の法則を提示したものである。

目　　次

Ⅰ．緒　　言

繼電器回路に於ける單部分路とは，繼電器の接點群と之等を接續する單純路素群とから構成されるもので(文獻 (1) 參照)， 其の包含する接點群と單純路素群との種々な組合によつて，實に數多くの幾何學的圖形が考へられる。又其の兩端から見たイムピーダンスが時間の經過につれて或は零に或は無限大に變遷して行く機能，即ち單部分路の作働機能は，同一幾何學的圖形に於ても，接點の時位の變更に依り，變化する事が多い事は，先に筆者の一人が發表した數例(2)に示す通りである。

併し乍ら之等の無數の單部分路の作働機能は總て悉く異るものではない。 傳送回路理論に於て driving-point impedance の等しい等價回路が存在すると同樣に，繼電器回路に於ても作働機能の等しい單部分路即ち筆者の謂ふ等價單部分路が存在するのである。單部分路の目的は其の作働機能の利用にあるので，與へられた單部分路の作働機能の見透を附け，或は他の都合よき單部分路で置換し，若しくはより簡單な單部分路を採つて經濟化する等の見地から，等價單部分路の一般的な理論を求めて置く事は極めて重要な問題と考へられる。

本稿は如上の見地からして，等價單部分路の變換に關する一般法則を提示して諸賢の御批判を仰がんとするものである。

單部分路の作働機能を求める場合に，接點の時位變換の影響を一般的に論ずる事は可成り面倒な問題となるので，玆に等價單部分路の變換理論を取扱ふに當つて，時位の影響を考慮に入れず單に幾何學的圖形として概括的に取扱ふものと更に時位の影響を併せ考へるものとの二種に分類し，本稿では先づ前者を論ずる事にする。實際的な立場からは後者まで論及しなければならないので，之は更に稿を改めて發表する心算である。此の點は諸賢の御諒承を御願ひ致し度く，本稿も其の心構へで御覽願ひ度いのである。

扨て本稿では，先づ任意の單部分路の代數的取扱に於て分配法則が成立する事を證明し，消去の法則を述べ，之等を用ひて直列ー並列の等價變換及びブリッヂ形式から直並列形式への等價變換を行ひ，更に一般に並列分解の法則を提示し，以上に依り單部分路の基本形式を見出し，最後に双逆關係にある單部分路を含む場合の等價變換の若干例を示したのである。

Ⅱ．分　配　の　法　則

先づ等價單部分路の變換理論の基礎として分配の法

The Theory of Equivalent Transformation of Simple Partial Paths in the Relay Circuit (Part I). Akira Nakashima, Member, Masao Hanzawa, Associate (Nippon Electric Co. Ltd.)
(J.I.T.T.E. December, 1936)

The first page of the paper

Nakashima, A., Hanzawa, M.

"The theory of equivalent transformation of simple partial paths in the relay circuit (Part 2)

Journal of the Institute of Telephone and Telegraph Engineers of Japan

No. 167, December 14, 1936, published February 1937.

論 文 欄

繼電器回路に於ける單部分路の等價變換の理論*（其の二）

會員 中嶋 章　准員 榛澤正男

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內容梗概

先に發表した拙稿"繼電器回路に於ける單部分路の等價變換の理論其の一"(1) を基礎として，更に單部分路の內部機構に關する等價變換の理論を展開し，若干の例を擧げて理論の說明を試みたものである。

目 次

Ⅰ. 緖 言

先に發表した拙稿"繼電器回路に於ける單部分路の等價變換の理論其の一"(1) では，幾つかの任意の單部分路から構成され居る合成單部分路に於て前者を單位として考へた等價變換の理論を述べた。換言すれば單位として考へた單部分路の內部機構には立入つて考へなかつたのである。即ち謂はゞ macroscopic な見方をしたのである。

斯かる見地に立つ前稿の理論はその一般普遍性を持ち形式的な美しさを有して居るが，實際問題に當つては更に單部分路の內部機構に關する等價變換の理論を考察する必要がある。單部分路の內部機構として特に主要な點はその含有する接點の型式と各接點の作働する時間的順序即ち接點の時位との二つであらう。

本稿では或る與へられた單部分路を斯くの如く謂はゞ microscopic な見地から眺めた場合の等價變換の理論を展開したものである。其の基礎としては前稿の一般理論を採るは勿論，又表現形式も同樣に代數的な取扱ひを行つた。

先づ單部分路の內部機構を，その含有する各接點の接點型式，接點の所屬する作働素，接點の時間的作働形態及び各接點間の接續形式等の因子を取入れて代數的に表示する方法を求め，次に接點型式及び時位に關する消去の法則を見出し，最後にかくして確立された理論を若干の場合に適用した例を示して說明する事とする。

Ⅱ. 單部分路の代數學的表示

先に發表した拙稿(1)では，幾つかの任意の單部分路から構成される合成單部分路を其の構成分たる各單部分路を單位として代數學的に數式を以て表示し得る事提を示し，加法及び乘法に關しては普通の初等代數學の場合と同樣に組合せ，交換及び分配の諸法則が成立する事を證明し，又單部分路に特有の消去の法則を求め，かくして從來は我々の頭の中で種々思索を廻らし謂はゞ算術的に考へたものを數式で以て代數的に簡便に取扱ひ得る事を見出したのである。此の數式的表示の型式としては或は又他の型式も種々考へられるであらうが，筆者の現在の考へでは拙稿(1)に提示したものが最も簡單で解り易いと思はれるので，敢て先に發表した次第である。

扨て今與へられた或る單部分路を表示せんとするに當つては其の含有する總ての各接點を取入れねばならぬが，各單一の接點も夫に附屬した單純路素を含めて見ると又一つの單部分路である。然るに上記の理論に於て着目した單部分路は全く任意のもので差支ないのであるから，今の場合も亦同樣の表示型式を採り得る理である。

次に今表示せんとする單部分路の內部機構を一般的に考へて見よう。玆に內部機構としては，單部分路の動幾何學的性質を支配する處の次の四種の因子を考へれば十分である。

i. 含有する各接點の型式
ii. 各接點の夫々所屬する作働素
iii. 各接點の時間的作働形態
iv. 接點群相互間を結合する單純路素群の接續形式

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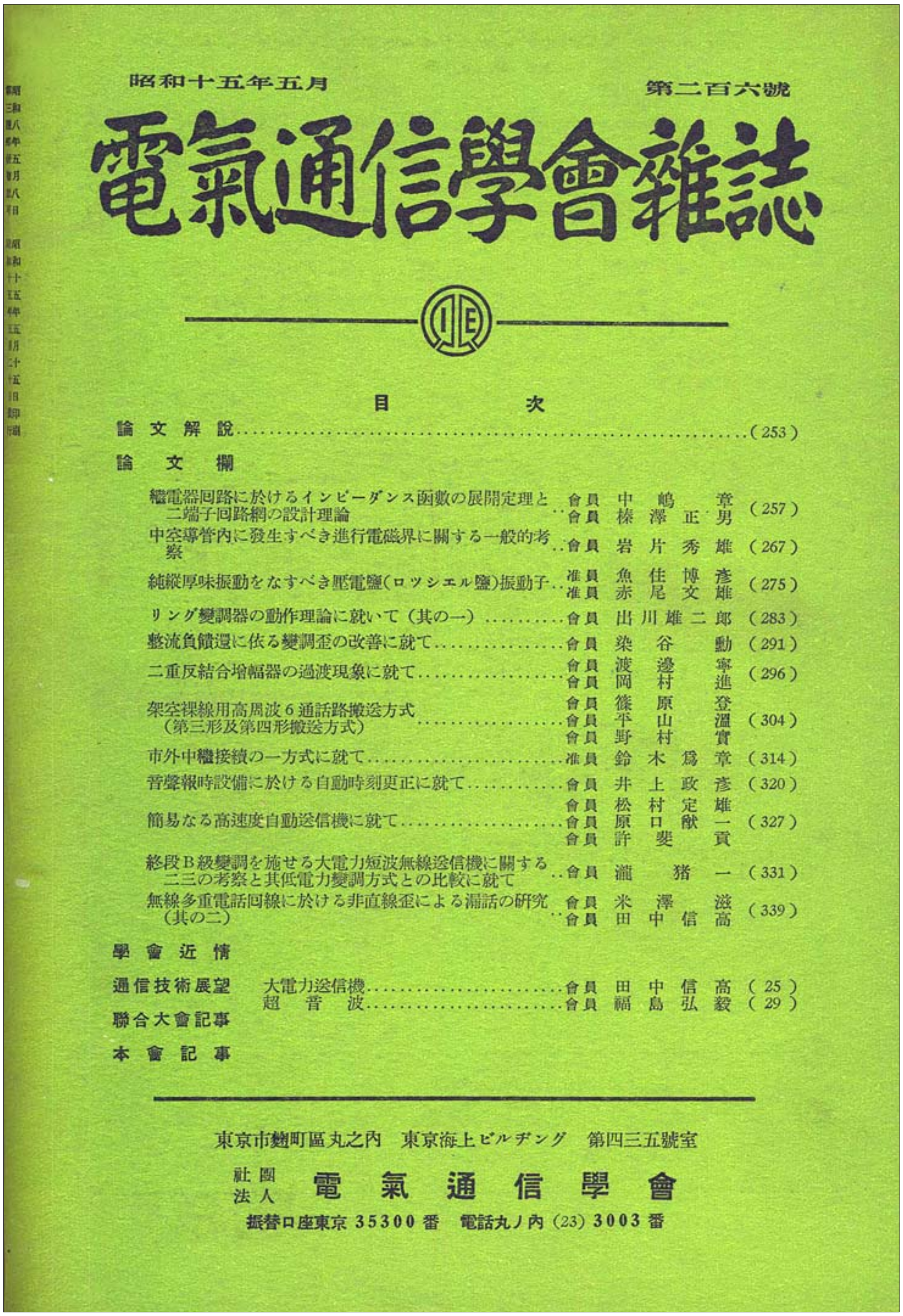
* The Theory of Equivalent Transformation of Simple Partial Paths in the Relay Circuit (part II).

Akira Nakashima, member, Masao Hanzawa associate (Nippon Electric Co.)

The cover with contents of
Journal of the Institute of Electrical Communication Engineers of Japan
No. 206, May 1940
The first paper is A. Nakashima's "The Law of Development of Impedance Functions and Theory of Designing Two-Terminal Networks in Relay Circuit."



昭和十五年五月　　第二百六號

電氣通信學會雜誌

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The first page of the paper
Nakashima, A., "Current status of
wired communication technologies in the United States"
Journal of the Institute of Electrical Communication Engineers of Japan
August, 1940, 477-484.

電氣通信學會雜誌第二百九號

昭和十五年八月

講　演　欄

米國に於ける有線通信技術の現狀

正員　中　嶋　　章

(日本電氣株式會社)

只今御紹介に與りました中嶋でございます。昨年の9月歐米の長距離通信の關係を調査して來るようにと云ふ社命に依りまして，日本を出ましてアメリカへ先づ參つたのでございますが，御承知のやうに昨年の秋歐洲に戰亂が勃發しまして，ヨーロツパへ參りましても何も私共の參考になるやうなものは見せて吳れないと云ふ實情でございましたので，殘念ながらヨーロツパへ參りませぬでアメリカだけを見まして此の四月に歸つて參りました。歸つて來ると匇々學會の方から何か話をしろと云ふ事で實は非常に困つたのであります。と云ふのは現在のやうな世界の情勢に於きましては，各國に於ける科學的な研究，色々な工業的の設備と云ふやうなものはそれぞれ外國に對して祕匿して居りまして，ドイツでは旣に數年前から實施して居るやうでございますが，アメリカに於きましても丁度昨年の秋から色々なスパイの取締が嚴重になり，一方政府の指令に依りまして新しい研究は外人に對して見せないと云ふやうな方針になりましたので，仲々思ふやうにうまい種も仕入れることが出來なかつたのであります。それからもう一つは皆樣御承知のやうにアメリカに於きまする通信關係のものは從來色々なペーパーに澤山載つて居ります。さう云ふ譯で特に目新しいことを話せと言はれると洵に實は困るのであります。所が翻つて現在の狀勢を考へて見ますと，此の非常時に於きまして貴重な國帑の一部を割いて戴いて見て參つた譯でございますから，多少なりとも御參考になればそれに對する義務が果せるのではないかと思ひまして，實は今日此の演壇に立つた譯であります。隨て大體皆樣の御承知のものでございますが，主として長距離通信のアメリカに於ける現在の狀況に付て御話申上げて見たいと思ふのであります。其の點一つ御含み置きの上御聽取を願ひたいと思ふのであります。

長距離通信の方面に於きましては，皆樣御承知のやうに廣帶域傳送方式，是が現在の通信技術界の花形でありまして，現にアメリカでやつて居りますものは數年前から研究をやつて居ります裸線の上に 12 チヤンネルの搬送電話方式を從來のC型の上に載つけようと云ふもの，之を J 型と言ひ，それからもう一つは我國に於て旣に逸早く研究が發表され而も實施されました無裝荷ケーブルを使つた搬送通信方式，それをアメリカでは K 型と呼んで居ります。それから更にコアキシヤル・ケーブルを使ひました搬送通信方式，此の 3 色あるのであります。

是等 3 種のことに關しては從來色々なペーパーに澤山研究結果が發表されて居りますが，現在どんな風に實施して居るかと申しますと，大體第一圖に示す通りであります。先づ無裝荷ケーブル用 12 チヤンネル搬送電話方式の K 型と致しましては，ニユーヨークからフイラデルフイアを通りましてワシントンへ行くもの，是は旣にペーパーに出て居りますが，現在實施して居りますシステム數はニユーヨークからワシントン間が 3 システム，それからニユーヨークからワシントンを通りましてノースカロライナ州にありますシヤルロツトへ行くものは 5 回線旣に實施して居りましたが，其の後 4 回線の追加工事を行ひ現在に於ては合計 9 回線にして居ります。それから北の方ではデトロイト，トロント間に 6 回線，デトロイトからサウスベンド間に 3 回線實施されて居ります。御承知のやうにアメリ

本稿は昭和 15 年 6 月 19 日 本會講演會に於ける速記である。

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通信技術展望

本欄は電氣通信工學界に於ける重要題目を各専門家が解説的に執筆せられたるものである。

繼電器回路の理論

正員　中嶋　章
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I. 緒　　言

繼電器回路を回路網として眺め夫の接續様式と作動機能との關係を究明して一つの理論體系を樹立し，一般的な基本性質を明かにして思索的勢力の經濟化を行ふ事は，從來の非科學的な取扱ひ方に鑑み極めて緊要な問題である。以下述べる處は筆者等が昭和 10 年以來本誌上に發表したものゝ要旨である。近年獨逸及び米國に於て一二の文獻が發表されたが其の理論的根據は筆者等の方法の踏襲に過ぎない。

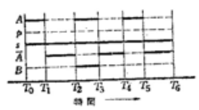
扨て回路網的理論を展開せんとする場合，受動網と能動網とに分類して論ずるのが便利である。玆に受動網とは接點と接點相互間を接續する導體（之を路素と呼ぶ）とから構成される回路網を謂ひ，本稿には受動網の理論のみを紹介する。何となれば受動網こそ繼電器回路網として最初に考究すべき基本的對象と信ずるからである。

II. 受動網の數學的取扱ひ方

一般に接點は時間の經過と共に或る時は閉ぢ或る時は開き，かくして受動網に於ては夫を構成する接點群の各時間的作動形態と之等の接點群をつなぐ路素群の接續様式とに依つて時間の經過と共に勢力の通路の圖形が飛躍的に種々に變遷する。從つて受動網中の或る點相互間の勢力の傳達に關して夫の有及び無の二つの狀態が時間の經過と共に飛躍的に且つ交互に生ずる。勢力傳達の有無をインピーダンスの觀念で考へると，インピーダンス値として零値及び無限大値の 2 種の極限値が時間の經過につれて交互に分布するのである。かくの如き 2 種のインピーダンス値の時間的分布の形態を一括して受動網のインピーダンスとして把握する。傳送回路網理論の受動網のインピーダンスは周波數對インピーダンス値の形態を指すのであるが，繼電器回路理論の受動網のインピーダンスは 0 及び ∞ インピーダンス値の時間的分布の形態を指すのである。

受動網のインピーダンスは各接點インピーダンスと之等の接點群をつなぐ路素群の接續様式とで支配され此のつなぎ方が重要な對象となつてゐる。從つて理論的研究を行ふには之等の 2 要素を併せて處理するのに適した數學的方法を先づ見出さねばならぬ。筆者等は種々研究の結果かゝるインピーダンスを一次點集合の理論[1]に依り解釋し集合論の演算形式を以て記號式演算を行つたのである。

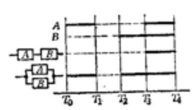
インピーダンスの集合論的解釋を第一圖に就て説明する。圖中實線及び點線はインピーダンス値の夫々零値及び無限大値を示す。一般にインピーダンスは A の如く表はされ，從つて無限大値のみか又は零値のみか何れか一方の値の時間的分布形態さへ定まれば自ら決まる。故に今無限大値のみに着目し考慮すべき全時間領域内に於て常に無限大値のみを呈するインピーダンス p を基礎に採り，之を總ての各時點に對應する無限大値點の總てを要素として持つ完全集合と考へる。任意のインピーダンス A は一般に p の部分集合，全時間領域に亘り常に零値を呈するインピーダンス s は空集合，A と逆のインピーダンス値分布を呈するもの（$\bar{A}$ と記す）は p に關する A の餘集合，又 A と B と比較すると A は B の部分集合である。



第　一　圖

二つのインピーダンス A と B と比較して，兩者が等しい集合の時 $A=B$ と表して等インピーダンス，A が B の餘集合の時 $A=\bar{B}$ と表し互に逆インピーダンス，A が B の部分集合なる時 $A<B$ と表し A は B より小なり，等と稱する。

次に任意の 2 個のインピーダンス A と B とを直列及び竝列に接續した場合を第二圖に就て考へる。圖からして直列接續の場合は集合の和，竝列接續の場合は集合の積に相當する事を知る。故に集合演算の記號として最も簡明な集合和の記號 +，集合積の記號 × を採用すれば，直列接續せるものゝインピーダンスは $A+B$ で表現され，竝列接續せるもの



第　二　圖

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は $A\times B$ で表現される。而して後述する如く如何に複雑な回路接續圖形に就ても受動能のインピーダンスに關する限り必ず直並列接續の樣式に歸一して取扱ひ得る故に，記號 + 及び × は演算記號であると同時に接續樣式をそのまゝ表現し得るのである。玆に集合算法の優れた特長があり筆者等の採用した所以である。

上記の説明では集合の要素として無限大インピーダンス値點を採つたが，零値點を要素として採つても集合算法で處理し得る事は云ふまでもない。唯兩者の相違は單にかゝる算法に存在する双對性に歸せしめられる事で，何れを採つても理論の一般性に變りはない。

集合論の算法は，命題算法或は論理の數學として19世紀中葉 G. Boole が創始し其の後發展せしめられた所謂ブール代數そのものであり，文獻[2][3][4]を參照され度い。(4) は手頃で簡明な良本である。

III. 演算原則と基礎關係式

1. 演算記號 + 及び × に就て夫々交換律及び結合律が成立すると共に，分配律としては下記の如き二つの關係が成立する。

$$A\times(B+C)=A\times B+A\times C$$
$$A+B\times C=(A+B)\times(A+C)$$

2. $A+A=A \qquad A\times A=A$

3. 大小の關係として次の關係式が成立する。但し論理的辭句"ならば"の代りに記號 → を用ふ。

$$s\leqq A\leqq p$$
$$A<B,\ B<C \ \rightarrow\ A<C$$
$$A+B\geqq A \qquad A\times B\leqq A$$
$$A<B \ \rightarrow\ A+B=B,\ A\times B=A$$

4. 逆インピーダンスの關係式

$$A+\bar{A}=p \qquad A\times\bar{A}=s$$

5. 逆變換の演算

$$\overline{(A+B+C)}=\bar{A}\times\bar{B}\times\bar{C}$$
$$\overline{(A\times B\times C)}=\bar{A}+\bar{B}+\bar{C}$$

等式に於ては兩邊を夫々逆變換するも等號關係は成立するが，不等式に於ては兩邊を夫々逆變換すると不等記號 < 又は > の向きが反對となる。

6. 双對の原理

或る一つの關係式が成立する場合には，その關係式に下記の置換を施して得る他の一つの關係式が必ず成立する。

$$\begin{pmatrix} + & \times & > & < & s & p \\ \times & + & < & > & p & s \end{pmatrix}$$

方程式論は玆に省略する。尚ほ上記の諸關係は，演算記號 + 及び × がそのまゝ接續樣式を示すものなる故直ちに回路網的意義を明示してゐる。卽ち等式は二端子回路網の等價變換を意味し，逆變換は逆二端子回路網を得べき最も簡單な例を示してゐる。尚ほ記號 × は必要なき限り便宜上以下之を略す。

IV. 二端子回路網の基礎理論

1. 二端子インピーダンスの表現法——重疊の理

一般に如何に複雜な接續圖形を持つ受動網に於ても夫の任意の2點から見た二端子インピーダンス Z は，該2點を結ぶ總ての通路の各二端子インピーダンス (Z_1, Z_2, ……, Z_n) の總積に等しい。但し總ての通路としては，1點より他の1點へ至るに受動網中の同一集合點は1度唯1度のみ過ぎる如き異る總ての通路を採る事が必要であり且つそれで十分である。卽ち

$$Z=Z_1\times Z_2\times\cdots\cdots\times Z_n$$

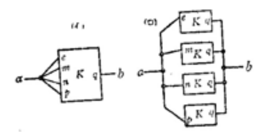
例へば第三圖に示す如き簡單な電橋回路では必要且つ十分な採るべき通路は圖示せる如く4個あり Z は，

$$Z=(A+C)(A+E+D)(B+E+C)(B+D)=AB+CD+ADE+BCE$$

第三圖

2. 並列分解又は並列結合

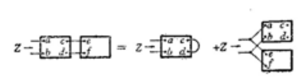
任意の二端子回路網の一方の端子に於て複數個の分岐せる枝路がある場合には，原二端子回路網に於て分岐枝路の夫々1個宛のみが存在する如き複數個の二端子回路網を總て並列に接續して構成される二端子回路網へ，原二端子回路網等價變換する事が出來る。例へば第四圖に示す通り (イ) は (ロ) へ等價變換される。此の等價性は可逆的で，(イ)から(ロ)への變換は主として解析的研究に，(ロ) から (イ) への變換は回路網合成の研究に夫々役立つ。



第四圖

3. 複合二端子回路網の分解

第五圖の左邊に示す如き任意の2個の受動網を2點で接續して合成された二端子回路網のインピーダンスに就ては右邊に示す樣に分解する事が出來る。此の理を押し進めて一般に多數の點で接續された複



第五圖

The first page of the paper
Nakashima, A., "Theory of relay circuit"
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No. 220, July 1941, 397-406.

電氣通信學會雜誌第二百二十號
昭和十六年七月

講演欄

繼電器回路の理論

正員　中嶋　章
(日本電氣會社)

1. まへがき

御承知のやうに繼電器回路と申しましても色々ございますが，最も代表的なのは自動電話交換方式でありまして，各國に於て既に立派な各種の方式が發展せしめられまして，實際に工學上の目的と致しましての實用性を十二分に發揮致して居ります。又自動電話交換方式の方で發達せしめられました所の種々の回路網は，その外の自動的な管理制御方式，色々な種類もございますが，その方面に實際に應用せられまして，その繼電器回路の特長をよく發揮致して居ります。即ち現在に於きまして，繼電器回路といふものは各方面に於きましてそれぞれ立派に實用になつて居ります。ところが，それを構成します所の繼電器回路に對する理論と申しますか，さういふものは殘念ながら見當らなかつたのであります。繼電器回路を構成しますためには，先づ普通經驗によりまする色々な定石を繋ぎ合せる，或はその定石を繋ぎ合せるやうな場合に於きましても繼電器の夫々の時間的の動作狀態を頭の中で考へつゝ，色々な接點形式の採り方を考量し，謂はゞ頭腦を酷使致しまして回路を設計して居るといふ狀態が多いやうに思ふのであります。つまり考へ方といふ點から見ると，非常な非科學的な方法を取つて居ると言つても過言ではないと思ふのであります。さういふ狀態を顧みまして何かその繼電器回路の基礎になるやうな理論はないか，若しさういふものがあれば從來繼電器回路の方面に從事して居る方々の頭腦の使ひ方の經濟化といふことが圖れるのではないか，或は繼電器回路と申しましてもそれ等の一般的な色々な性質といふものはもつとはつきりと究明せられまして，その結果この方面に携はる人々に何等かの助けになるやうなことが見付かるのではないか，さういふものは是非とも必要ではないかといふことを考へまして，主として繼電器回路の回路網に關する基礎的な色々の性質或はその理論といふものを少し突ついて見ました。さうして昭和 10 年から當學會雜誌の貴重な紙面を汚さして戴きまして屢々私の考へました事柄を發表させて戴いて居つたのであります。その概要をこゝでお話申上げまして何等かの御參考にもなればと存じて居る次第であります。

先程申上げましたやうに，私の研究の對象として主として取上げましたものは，回路網的に見ました繼電器回路の問題であります。繼電器回路の問題と致しましてはその外に過渡現象の問題とか色々ありませうが，これを回路網として取上げた場合を調べて見たのであります。これを回路網として眺めました場合に，先づ最も簡單な場合を申上げますと，先づ或る勢力源がありまして，さうして多くの接點とそれを繋ぐ所の導體から成る回路網があり，その次に之等に依つて制御される所謂繼電器がある。或はこの制御される繼電器の接點が更に元の回路網の中に入つて居るとか，又これ等の回路網によつてコントロールされる所の接點を以て構成させられる所の他の色々な繼電器の回路網が相互に關聯性を持つてゐるとか，色々な場合がございますが，研究して行きます一番根柢としましては，先づ繼電器の接點と，夫等を繋ぐ導體とのみから成る回路網を先づ最初に突つくのが一番必要ではないか，斯う考へまして，これを受動網と名付けました。それに對して能動網と名付けますものは，接點と斯ういふ導體以外に，更にこれによつてコントロールされる所の繼電器と勢力源をも考へに入れたものを稱する，さういふ大別方針によりまして，先づ受動網の關係を主として突ついて見たのであります。これからお話申上げま

本稿は昭和 16 年 4 月 26 日本會第 15 回通常總會に於ける講演の速記ある。

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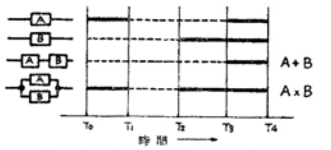
Journal of the Institute of Electrical Communication Engineers of Japan
No. 220, July 1941, 397-406.

ンス値を示すものは，これは s で現はして居りますが零集合になるわけです。それから A に對して丁度逆の時間的なスペクトラム圖形を成すもの（第一圖 $\bar{A}$）はこの p といふ完全集合に對して A の補集合といふものになる。さうして A の外に一つ B といふインピーダンスを考へますと，これは A よりも無限大インピーダンス値を餘計含んで居るのであります。隨て集合論的に見ますと，B は A よりも大である。斯ういふ考へ方を致しまして，總て受動網に於ける所のインピーダンスを點集合の理論によつて先づ解釋する。さうしますと，例へば任意のインピーダンス A と B が直列に接續されたやうな場合，或は並列に接續されたやうな場合はどんな風に表現されるであらうかを考へて見ます。第二圖に示しましたやうに，假に A, B



第二圖　直並列接續の記號表現

なる二種類の任意のインピーダンスが圖のやうな時間的形態を取るものと致しますと，直列に接續された場合は明かに斯うなる，並列に接續された場合は明かに斯うなる，これは極めて簡單な事柄でございますが，今これを集合論的に考へて見ますと，直列になりました場合には A の含んで居る集合の要素と B の含んで居る集合の要素とを兩方共に含んで居るのであります。ですからこれは集合の和と解釋して行くわけであります。それから並列に繋がつて居ります場合には，A と B とが共通に持つ集合の要素だけを持つて居る。隨てこれは集合の考へ方から言ひますと集合積として考へられる。斯ういふ風に受動網のインピーダンスといふものを集合の考へ方から行きますと，少くとも直列或は並列に接續されたものは集合演算の記號で現はし得るのであります。こゝに集合の和を＋，集合の積を × で現はしてありますが，これは一番ポピュラーな符號を使つたに過ぎないのでありまして，この外數學の專門的な方面で使はれて居りますどんな符號を使つてもいゝのでございます。斯ういふ風にして行けば受動網の中に包含される各種のインピーダンスとその接續圖形に關して ＋ 及び × 記號で總て表現することが出來るのであります。

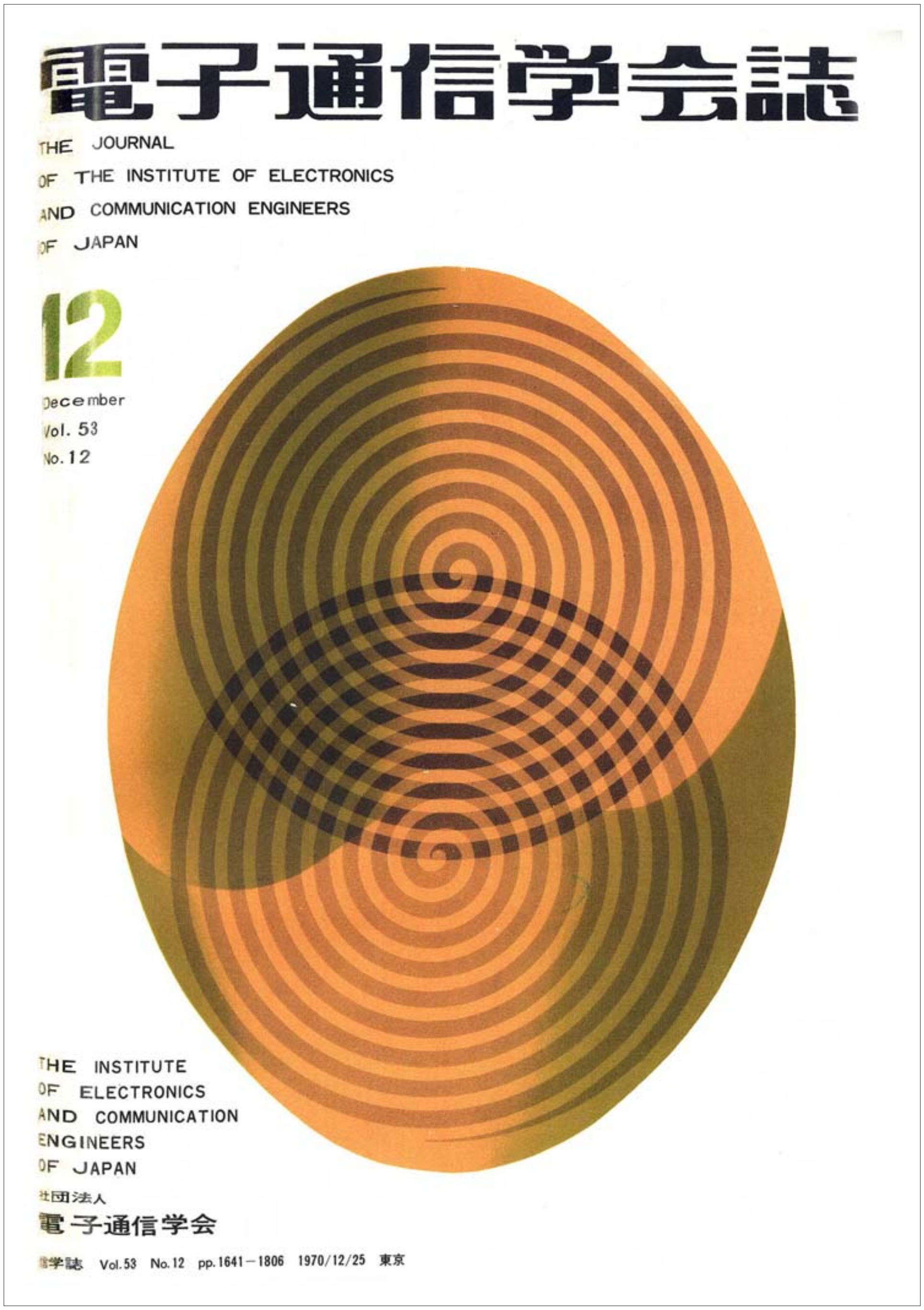
3.　演算原則と基礎關係式

然らば，斯ういふ演算をして行きます場合に，どんな數學上の演算原則によつてやつて行けばいゝかと申しますと，＋ 及び × の二つの記號に對しまして結合の法則及び交換の法則といふものは第三圖 (1) 及び (2) に示す樣に明かに成立します。それ以外に分配の法則と致しまして第三圖 (3) (4) に示しましたやうなものが成立する。(3) の式は普通の代數の通りであります (4) の式は普通の初等代數と少し違ふ。それから同じものが二つあつた場合，それの和及び積は夫々 (5) 式に示す樣にそのものに等しい。先程說明申上げましたやうな大小の關係に就きましては，A は B より小さく，更に B は C より小さければ (6 式) A は C より小さい。それから任意のインピーダンス A と B との和はその中の一つのインピーダンスに等しいか，より大きい。掛けたものはその中の一つに等しいか，より小さい。斯ういふ關係 (7 式) がある。それから若し大小の關係が存在します場合には (8 式)，A が B より小さい場合には $A+B$ は B になつてしまふし $A\times B$ は A になる。斯ういふ大小關係の存在します場合の演算結果は一種の消去則が行はれて居る恰好になつて居りまして，このために後の方の一般の演算が初等代數と相當違ふ所を生じて來て居ります。それから一つのインピーダンスとそれに對する逆のインピーダンス，つまり共軛關係に相當するものを A に對して $\bar{A}$ で現はしますと，加へたものは完全集合 p になる，掛けたものは s になる (9 式)，これは明かな話であります。それから $A+B+C$ といふものの逆のインピーダンスといふものをやはりこのバーで現はしますと (10) に示す樣な恰好になる。逆に掛ける印で結合させましたものの逆のインピーダンスは (11) 式に示す樣な恰好になる。これは數學の方で申しますと De Morgan の法則に相當するのでありますが，斯ういふものが成立する。又等號及び不等號で結ばれた關係の逆變換に就ては (12) 式が成立する。斯ういふ一連の演算の基礎原則といふものがありまして，それ等に對して一貫して成立する所の一つの原則がある。それは最後に (13) として記しました所の双對性による關係 Dual Principle でございまして，或る成立する式に於きまして上の方に書きました記號をそれぞれ對應する下方

1. $A+(B+C)=(A+B)+C$
   $A\times(B\times C)=(A\times B)\times C$
2. $A+B=B+A \quad A\times B=B\times A$
3. $A\times(B+C)=A\times B+A\times C$
4. $A+B\times C=(A+B)\times(A+C)$
5. $A+A=A \quad A\times A=A$
6. $A<B,\ B<C \longrightarrow A<C$
7. $A+B\geqq A \quad A\times B\leqq A$
8. $A<B \longrightarrow A+B=B,\ A\times B=A$
9. $A+\bar{A}=p \quad A\times\bar{A}=s$
10. $\overline{(A+B+C)}=\bar{A}\times\bar{B}\times\bar{C}$
11. $\overline{(A\times B\times C)}=\bar{A}+\bar{B}+\bar{C}$
12. $A=B \longrightarrow \bar{A}=\bar{B}$
    $A<B \longrightarrow \bar{A}>\bar{B}$
13. 双對性 $\begin{pmatrix} + & \times & > & < & s, & p \\ \times & + & < & > & p, & s \end{pmatrix}$

第三圖　演算原則と基礎關係式

The Japanese cover of
Journal of the Institute of Electronic Communication Engineers of Japan
Vol. 35, No. 12, December 1970
"Frontier Days in Retrospect" which includes A. Nakashima's paper
"Reminiscences of the Switching Network Theory", 1658-1661.


電子通信学会誌
THE JOURNAL
OF THE INSTITUTE OF ELECTRONICS
AND COMMUNICATION ENGINEERS
OF JAPAN
12
December
Vol. 53
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THE INSTITUTE
OF ELECTRONICS
AND COMMUNICATION
ENGINEERS
OF JAPAN
社団法人
電子通信学会
信学誌 Vol.53 No.12 pp.1641－1806 1970/12/25 東京

The Japanese contents of
Journal of the Institute of Electronic Communication Engineers of Japan
Vol. 35, No. 12, December 1970
”Frontier Days in Retrospect” which includes A. Nakashima’s paper
”Reminiscences of the Switching Network Theory”, 1658-1661.

電子通信学会誌

53巻12号(昭和45年12月) 目次

The English contents of

Journal of the Institute of Electronic Communication Engineers of Japan

Vol. 35, No. 12, December 1970

”Frontier Days in Retrospect” which includes A. Nakashima’s paper ”Reminiscences of the Switching Network Theory”, 1658-1661.

Jour. Inst. Electronics Comm. Engrs, Japan

Vol. 53, No. 12, December 1970

CONTENTS

The first page of the paper
Nakashima, A.
"Reminiscences of the Switching Network Theory"
Journal of the Institute of Electronic Communication Engineers of Japan
December 1970, 1658-1661.

UDC 621.372.06(0：82-4)

フロンティアを求めて

スイッチング回路網理論の思い出

中嶋　章

中嶋　章：安藤電気株式会社
Reminiscences of the Switching Network Theory. By Akira Nakashima, Member (Ando Electric Co., Tokyo).
資料番号：昭 45-182 [フロンティアを求めて-5]

1. ま え が き

このたび編集委員の方から，スイッチング理論についてのエピソードなど思い出ばなしを書いてみないかとのおすすめがあった．

私が手を染めたスイッチング回路網の理論は，もうすでに 30 年以上も昔のかび臭いものであり，また当時若年客気のいたりでその所論もきわめて生硬かつ未熟のもので誠にはずかしく，特にその後わが国ならびに諸外国で立派な論文や著作が発表刊行されているので，おすすめを素直に引き受けてよいものかどうかと，30 年にわたる長いかつ変化の多い幾重もの厚い緞帳が急に一度に引き上げられて遠い過去に直面させられた想いで，しばらくはとまどった．

しかしながら，スイッチング回路網理論は私の 20 才代の若い情熱をもやして一途に取り組んだ課題でもあり，他目にはいかに拙く見えようとも私の生涯の一つの記念碑でもある．また当時格別のご激励をいただいた恩師諸先輩や学会のご厚情，上司で終始変わらぬご指導をいただいた丹羽保次郎博士や協力者たる同僚榛沢正男氏らへの感謝の念にも駆られ，あるいは時あたかも第二次大戦が欧州でぼっ発し緊迫感とあわただしさに包まれていた米国で B.T.L. の Molina 博士らとの discussion や若き日の Shannon 氏との出合いなども次第に思い出されてきて，もし拙稿が若い人達に少しでもお役に立てば幸いと，おすすめに従って拙い筆を執ることにした次第である．

2. 研 究 の 動 機

まず最初に，なぜ私がスイッチング回路網理論に手をつける気になったのか，その動機についてお話ししいたしたいと思う．そのため私の置かれた環境，当時のスイッチング回路に関する学問的レベル，それらによって私の心中に探求心が誘発された状況などを記したい．

私は昭和5年，大学の電気工学科を終えると直ちに日本電気に入社，技師長丹羽保次郎博士のご指示で研究係の中の嶋津保次郎氏ご担当の特殊リレー回路装置研究班に配属された．この班では，嶋津氏の天才的なリレー回路方式に関する諸発明と指導の下に，発変電所の遠方監視制御装置や新しい自動交換方式などの研究開発が行なわれていた．

最初のうちは，複雑な機能を要求されるリレー回路をスイスイと考え出して行かれる嶋津氏の霊力にただ感嘆するばかりで，私は何のお役にも立たなかった．

そのうちに，リレー回路装置としての具体的な実施面の問題として，装置機能の誤動作を防止し確率的な信頼性を向上するため，リレー回路内の過渡現象を究明しなければならぬことを知ったが，当時は特にリレー回路機能に関連しての過渡現象論はほとんど文献もなくまた一般の関心も薄かった．そこで一般過渡現象理論を具体的な各種のリレー回路に適用して，リレー回路設計者の心得るべき諸現象を究明することに努め，当時の社誌「日電月報」などに連載しリレー回路

Cover page with the contents of
the *Institute of Electronics and Communication Engineers*,
Vol. 54, No. 3, March 1971, with
the condolence on Nakashima's death written by
Zen-iti Kiyasu, Kan-ichi Ohhashi and Yasujiro Shimazu.

Jour. Inst. Electronics Comm. Engrs, Japan

Vol. 54, No. 3, March 1971

CONTENTS

The first page of the condolence on Nakashima's death written by Zen-iti Kiyasu, Kan-ichi Ohhashi and Yasujiro Shimazu and published in the *Institute of Electronics and Communication Engineers*, March 1971.

中　嶋　　章
1908.1.5-1970.10.29

中嶋章博士の訃に接して業績をおもう

喜安善市・大橋幹一・嶋津保次郎

1970年10月29日中嶋章博士は逝去された．生前における博士の論理回路に関する学術的業績は，本会として世界に誇る第一級のものである．このことを会員とともに想起して博士の冥福を祈るため，大橋幹一，嶋津保次郎の両名誉員をわずらわして，追悼的小論をお願いし，本号に掲載することにした．

司馬遼太郎によれば"ひとつの時代がすぎ去るというのは，その時代を構築していた諸条件が消えるということであろう．消えてしまえば，過ぎ去った時代の理解というのは，後の世の者にとっては同時代の外国に対する理解よりもむずかしい"．計算機や交換機の基礎としての論理回路の知識の普及した現在からは，中嶋博士の研究の苦心は理解しにくいかもしれない．昨年本誌12月号の"フロンティアを求めて"に中嶋博士の"思い出"がまにあったのは誠に幸であった．これは絶筆に近い．本号の二小論とこの中嶋博士の稿によって，博士をしのび，かつ戦前の論理回路の夜明けの姿を後世に伝えんとする次第である．

（喜安善市：本会編集長　岩崎通信機株式会社）

中嶋　章氏の業績

私が電気通信学会の編集幹事をしていたころ，学会講演を誰に依頼するかにつき考えあぐんでいたときに丹羽保次郎博士から中嶋章氏の継電回路のことはどうだろうとご推薦があった．

私自身は主として伝送関係に理解があり，学会の論文なども伝送関係，詳しく言えば搬送，無線そして電気回路網などに接する機会が多く，交換関係の論文などは当時ほとんどなかった時代である．

交換関係は，震災復旧のころは大変はなやかであったがその後はそれほどではなかったし，論文や講演としては基本的なものが少なくて読みづらいし，聞きづらくもあった．

そういう状態であったから丹羽博士のお話に対しても乗り気のない返事をしたが，同博士から日電月報を送っていただいて拝見した．

それは，中嶋章氏あるいは中嶋，榛沢両氏連名の数回にわたる接点回路の理論であった．Boole代数の見事な応用で完全に魅せられてしまった．命題算というのには私は初めてお目にかかった．岡田幸雄氏などはたいへん高く評価しておられた．

私は交換の技術に理論がなかったとは言わない．少なくとも，トラヒックの方面にはいろいろの確率的な理論が応用されていたが，その他は多く経験的なもののように考えられた．

私自身も，中嶋氏の論文に刺激せられて数年後にこの方面のことに多少頭をつっこんだことがあったが，多数の人々がこういうことをやってゆく時代ではなかった．

中嶋章博士はこれまで種々の貢献をなされた人ではあるが，私には上述のことが一番印象に残っておる．

その後20～30年間，この方面の研究は内外の学界においてもぽつりぽつり散見する程度にすぎなかったが，電子計算機の出現，特に半導体によるディジタル電子計算機の発達に歩調をあわせて，この方面の研究は大いに受け継がれてきている．

中嶋博士が，近年主として経営的な仕事に携わられ

3 Nippon Electrical Communication Engineering

English versions of papers by Nakashima and Nakashima and Hanzawa have been published in the journal *Nippon Electrical Communication Engineering* a publication of the *Institute of Telegraph & Telephone Engineers of Japan* (**Densin-Denwa-Gakkwai**), Tokyo, Japan.

The cover page, the contents, and the foreword of the first issue of this journal published in September 1935 are shown at pages 77, 79 and 81.

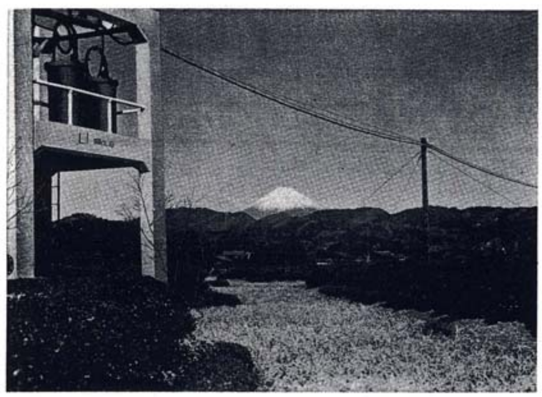
The cover page of *Nippon Electrical Communication Engineering* issue No. 9, February 1938, containing the paper enumerated as item 6 in the list of publications by Nakashima and Nakashima and Hanzawa above is shown at page 87. The page with contributions to this issue with biographies and photos of Nakashima and Hanzawa is shown at page 89.

The cover page of the first issue of
Nippon Electrical Communication Engineering
No. 1, September 1935.

日本電氣通信工學

NIPPON ELECTRICAL COMMUNICATION ENGINEERING

No. 1 Quarterly Edition September, 1935



Densin-Denwa-Gakkwai

(Institute of Telegraph & Telephone Engineers of Japan)

Tokyo, Japan

The contents page of the first issue of
Nippon Electrical Communication Engineering
No. 1, September 1935.

NIPPON
ELECTRICAL COMMUNICATION ENGINEERING

No. 1, September, 1935

— CONTENTS —

The foreword by **T. Akiyama**,
the President of the *Institute of Telegraph & Telephone Engineers of Japan*,
in the first issue of
Nippon Electrical Communication Engineering
No. 1, September 1935.

FOREWORD

During recent years the electrical communications business in Japan has developed phenomenelly, and the present satisfactory state of affairs is undoubtedly due to the efforts in research, manufacturing and practical fields having been conducted in close harmony, and being based on a solid foundation of applied science.

As evidence of this, today the number of telephones in the country has reached nearly one million, whereas as many as two million homes have the advantage of radio broadcast receiving sets. Similarly external communication by radio telephone is possible between Japan and the majority of the major countries of the world.

As a result of these favourable circumstances, the Institute of Telephone and Telegraph Engineers of Japan is interesting itself in all the important phases of the electrical communications art. This Institute is enjoying full prosperity with 5,000 members, and has the wholehearted recognition of the public as an institution of authority in this field of science.

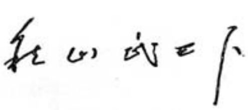
The monthly journal published by the Institute has been presenting an excellent series of articles fully covering the field of communications, written by both executive and technical specialists, and has thus been a powerful stimulant to activity in the Japanese telephone, telegraph, and radio worlds.

For some time past there has been a marked desire on the part of engineers in foreign countries to learn through this journal of ours, more of the general trend of communications in Japan, and the characteristic features of our studies and improvements in the art, but owing to the difference of language they have been severely handicapped. We are determined to make possible a wider circulation of our journal in foreign countries by reprinting periodically in English.

The state of technique attained by us Japanese has largely been obtained from abroad in the past, and it is needless to say our duty is to bring to the notice of foreign countries our own achievements in research, and thus to repay the friendship shown to us through many years. By so doing it is hoped that we may also contribute to the total advance of world civilization.

Similarly we feel a sence of responsibility towards our colleagues who have contributed valuable reports of their studies to our journal, for it is not desired to confine the benefits of their work to within our own country, but by publishing this English edition to open the door to wider spheres.

I sincerely hope that the future of this English edition will be crowned with success and prosperity.



T. Akiyama

President
Institute of Telegraph & Telephone Engineers of Japan.

The page in
the *Journal of the Institute of Telegraph & Telephone Engineers of Japan*
with an announcement of the sale of the first issue of
Nippon Electrical Communication Engineering
No. 1, September 1935.
It says the price of a copy is one-yen (fifty-sent for members).

英 文 雜 誌

NIPPON ELECTRICAL COMMUNICATION ENGINEERING

第 一 號 發 刊
購 讀 募 集

近年本邦學術工業の進歩は誠に顯著なるものあるも之を海外に紹介する途に於て缺くる所あり，我國工業品の海外進出に當りても我國技術に對する信頼を贏ち得る上に於て，歐米諸國に比し多大の不利を招き居るは頗る遺憾に堪へざる次第なり。此は徒らに浮薄な廣告宣傳に努むるより寧ろ我國技術の現狀に對する海外諸國の認識を深むる事も肝要にして，本學會の使命の一も亦玆に在るを感じ，今般多大の犧牲を拂ひ玆に英文雜誌 Nippon Electrical Communication Engineering 第一號を發刊するに至れり。

本誌は年四回發行にして，論文は本誌電信電話學會雜誌中より上記目的に適當する論文を選擇して掲載する外，各號斯界の權威者を煩はして本邦電氣通信事業隆盛の現狀を海外に紹介する事とせり。從て本誌は海外一般讀者のみならず，內地會員に對しても誠に有益なる參考資料なりと信す。

第一號の內容は下記の通りにして，一般希望の向には下記實費を以て頒布す。

實 費 金 壹 圓（本會々員に限り**金五十錢**）

NIPPON ELECTRICAL COMMUNICATION ENGINEERING

No. 1, September, 1935

— CONTENTS —

The second page in
the *Journal of the Institute of Telegraph & Telephone Engineers of Japan*
of an announcement of the sale of the first issue of
Nippon Electrical Communication Engineering
No. 1, September 1935 with the contents continued.

PART I

PART II (Abstract)

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The cover page of
Nippon Electrical Communication Engineering
No. 9, February 1938.

日本電氣通信工學

No. 9 February, 1938

NIPPON ELECTRICAL COMMUNICATION ENGINEERING

Quarterly Edition

Denki-Tūsin-Gakkwai

(The Institute of Electrical Communication Engineers of Japan)

Tokyo, Japan

The page with **Contributors to This Issue** of
Nippon Electrical Communication Engineering
No. 9, February 1938.

Mr. Akira Nakasima

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THE THEORY OF RELAY CIRCUIT COMPOSITION

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CONTENT.

SYNOPSIS.

This is a general discussion of and a systematic consideration on the composition of so-called relay circuit system, which has made surprising advancement in recent time in connection with automatic telephone exchange, remote control systems, etc.

It shows the fundamental idea and characteristics of the relay circuit, some of the interesting properties and theorems regarding the dynamic geometrical character, analytical treatments of simple cases, forms of relays, and then some of the fundamental systems of relay circuit composition.

It is noted that transient phenomena which arise inevitably in the relay circuit are not, however, included in this discussion.

I. GENERAL ESSENCE OF RELAY CIRCUIT.

1.1. Fundamental idea and definitions.

The relay circuits now in use are many in kinds and complex in variation. However, under general survey, the following definitions may briefly be given:

Relay Circuit is a method in which it becomes a mediator between some given phenomena and the corresponding desired phenomena, and by the use of relays as its composite elements, the occurrence of the former realizes the latter.

Next, taking these relays as its composite element in broad sense:

The relays may be defined as an element that determines, by presence or absence of its receiving energy, whether another energy is transmitted or not. In regard to energy the former is called controlling and the latter controlled energy. They are, however, termed merely with respect to one certain relay; and one energy may become sometimes the former and other times the latter. Thus, from this definition we find that the relay consists, in one part, of receiving controlling energy to determine its action, and in another part, of controlling directly the transmission of controlled energy. The former may be called acting element and the latter contact point. Contact point may be said as a composite element of transmitting path of controlled energy, which is controlled at its point. It is also made of a number of contact elements that have definite meaning in regard to mechanical contact point.

Accordingly, in the relay circuit, the energy transmitting path is made in general of energy source, acting elements, contact points, and of path element which is a part of the path that connects all of these. The path element containing no impedance against controlled energy is called simple

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path element, and that containing impedance complex path element. The relay circuit is therefore a combined system of these energy transmitting paths.

When the contact points complete operation for the desired condition after receiving controlling energy, and when the extinction of the controlling energy causes the contact points to return to their former position, they are respectively called operating and releasing, and they in combination may be termed acting.

Let us now consider the variation of energy in the relay itself. The usual controlling force received by the relay is necessary to be changed to various kinds of suitable energy to bring the contact points to operate, and in ordinary cases the contact points are given energy to resist the controlling energy. The former is called the mediating energy, and the latter the resisting energy. To function the mediating energy efficiently, a path is necessary. This is called the mediating path of energy.

1.2. Relay.

From the forgoing definition, a great many kinds of relays exist in relation with the following points:

a. Mutual relationship between the kinds and quantity of controlling and controlled energy.
b. Selective action regarding attribute of controlling energy.
c. Controlling form of transmission of controlled energy.
d. Time relation between reception of controlling energy and the transmission of controlled energy.
e. Type of construction of acting elements and contact points.

However, in general, its characteristics may be classified into the following 5 kinds:

Characteristic 1. Irreversibility of function.
〃 2. Discontinuity of function.
〃 3. Conversion or renewal of energy.
〃 4. Inevitability of transient phenomenon.
〃 5. Complexity of transmission paths of energy.

Irreversibility of function means that it is not possible to interchange the relation of controlling and controlled energy; that is, the law of reciprocity in case of negative networks cannot hold here. This may be quite apparent when we consider that relay is a kind of positive network. Discontinuity of function means that the relay starts to act only when the quantity of controlling energy becomes either above or below a certain value and the transmission of controlled energy is discontinuously controlled. Conversion or renewal of energy is an inherent nature of relays. Inevitability of transient phenomenon is a result accompained by the discontinuous function and is therefore one of the important elements in connection with time-action characteristics. The transmission path of energy usually exist in two or more paths, due to presence of acting elements and contact points, and may be considered as network of four or more multiterminals. These above mentioned characteristics are destined to cause the relay circuit, consisting of these elements, to have its peculiar characteristics.

1.3. Characteristics of relay circuit.

Relay circuit is in general a combined system of energy transmission paths made up of energy source, acting elements, contact points, and path elements. With different kinds of these composite elements and by connecting them in many ways, various relay circuits are produced, but there exist characteristics common to all of these circuits, some of which are not found in so-called transmission circuit, and these characteristics are quite interesting to note. They may be listed as follows:

Characteristic 1. Irreversibility of circuit function.
〃 2. Discontinuous recurrence of phenomena.
〃 3. Conversion or renewal of energy.
〃 4. Transient phenomenon.
〃 5. Dynamic geometrical character of circuit function.
〃 6. Presence of equivalent circuits (Multiplicity of solution).
〃 7. Capability of converting forms of phenomena.
〃 8. Capability of storing forms of phenomena.

Characteristic 1. Irreversibility of circuit function depends upon irreversibility of relays, the composite elements, themselves, and a series of phenomena which arise in a relay circuit always progress irreversibly.

Characteristic 2. Discontinuous recurrence of phenomena means that phenomena arising within the relay circuit, recur discontinuously both in quantity and in time, and is a peculiarity which does not exist in transmission circuits.

Characteristic 4. The relay circuit seems as though it is a large group of transient phenomena. These transient phenomena are very important elements that determine the time-relation of discontinuous phenomena; and without thorough understanding and observation on these, the rational application of relay circuits is impossible and its development may be also beyond the hope of realization.

The advantages and disadvantages of these transient phenomena are entirely under the control of functions of relay circuit, and it is common to note that disadvantages in some occasions may become advantageous in other occasions. In short, the transient phenomenon ought to be used or controlled freely by investigating its underlying truth.

Characteristic 5. Dynamic geometrical character of circuit function may be explained as follows: For the time being disregarding conversion of energy and transient phenomena in a relay circuit, and considering only transmitting path of energy, it becomes a geometrical figure. But this figure is not fixed, and changes its form discontinuously from time to time. From the time of occurrence of a given phenomenon until the time of obtaining desired phenomenon, various path elements, changing in their process of operation many times their combinations, form different geometrical figures step by step. This characteristic is also a peculiarity of relay circuits, and for this reason in dealing with relay circuits, it is necessary to consider one more dimension with respect of time added to two dimensions of geometric figures. Here exist the charm and complexity of relay circuits.

Characteristic 6. Presence of equivalent circuit (multiplicity of solution) means that relay circuits generally have many ways of realizing desired phenomena by given phenomena; in other words, there are many solutions to satisfy the desired conditions. This depends upon the presence of forms of equivalent circuits in relay circuits; and while equivalence of transmission circuits is considered as function of impedance, the equivalence of relay circuits is considered with mutual relationship between given controlling energy and contact points.

Characteristic 7. Capability of converting forms of phenomena means that in relay circuit it is capable, for instance, of interchanging mutually group phenomena with single phenomenon, and of converting the number of occurrence of phenomena. Such conversion of forms of phenomena is peculiar characteristics of relay circuits.

Characteristic 8. Capability of maintaining forms of phenomena means that it is capable of storing the given forms of phenomena within the relay circuit, when several given phenomena occurring at the same time, yet unable to realize more than one desired phenomenon at once, are required to wait in turn to function, or when it is necessary to store the given phenomena for a moment to prepare for converting the forgoing forms of phenomena, or when it is necessary to accumulate the successively occurring phenomena. This characteristic is another peculiarity of relay circuits and together with the characteristic of conversion is very important.

1.4. Gradation of function in relay circuit.

In general relay circuits, from the definitions given above, considering from their internal functions, are made up of the following three stages:

Initial Stage: The stage to convert the given phenomena to be suitable to the relay circuit, the mediator, and to introduce them into the circuit.

Intermediate Stage: The stage to treat the introduced form of phenomena to be suitable for the purpos of the circuit.

Final Stage: The stage to convert the resultant form of phenomena treated in the intermediate step into the desired phenomena.

In the circuit for initial stage, the problem is the function of relay itself due to the function of selective or energy conversion. In the intermediate stage, it is mainly necessary to make dynamic geometrical study of time-action characteristic and of combination of contact points and path elements. While in protective relay systems of heavy current field, the function of the initial stage is taken as the main importance, and intermediate stage circuit seems entirely eliminated, the automatic telephone exchange systems and automatic control systems have been dependent upon the phenomenal development of this intermediate stage circuit. The circuit development discussed later in this article is dealt mainly with such intermediate stage circuit, and detailed discussion of relay itself is not given.

1.5. Consideration of electromagnetic relay circuit.

The foregoing items are general essence common to relay circuits defined in section 1.1. As it is clear from the definition of relays, among relays there are not only mechanical structures, but also balanced bridge circuit, light bulbs, vacuum tubes, neon tubes, thyratron, measuring instruments, thermal relay, static relay, motors and generators. Of these light bulb, vacuum tubes, and thyratron each make up interesting relay circuit, but they are mainly of functions of initial stage, and therefore, as they do not possess dynamic geometrical interest, they are not discussed in this article. On the other hand, the circuit using electromagnetic relays, in a narrower sense, makes up most of the relay circuits, and its development in dynamic geometry has been so prominent that its application has been enlarged to a great extent. Consequently it will be discussed in detail here.

In the electromagnetic relays, action element itself is capable of storing magnetic energy, so that it inherently can not escape from transient phenomena, and together with dynamic geometrical change of the energy transmission path and the impedance characteristics of each path element, presents complex transient phenomena. Moreover, added to these are transient phenomena of dynamic nature in the mechanical contact points and transient phenomena caused by the changing magnetic path accompanied by their motion. These become principal causes that govern the dynamic transient nature of the circuit.

One action element is possible to have many contact points which are composed of various combinations of opening and closing as their basis, and this evidently is the reason for making the geometric figures of the circuit more complex and confusing.

The capability of holding the operative conditions due to the nature of resistive energy or by suitably connecting contact points, path elements, and action elements shows capability of storing the forms of phenomena; and the suitable combination of contact points and path elements or the presence of retarding time-action characteristic enables the forms of phenomena to be converted.

II. ON ACTION ELEMENT.

2.1. Substance of action elements.

The element of action is, as defined in section 1.1, a part of relay and it determines the action of relay by receiving controlling energy. As for the energies affecting this determination in electromagnetic relay there are magnetic energy, or mediating energy which is converted from controlling energy, and resisting energy which is supplied by mechanically movable parts that control directly the movements of contact points; and thus, action is determined as the result of mutual check of these energies. It is constructed with coil (it may be called action coil) that receives directly the controlling energy, magnetic path, and mechanically movable parts; and especially in order to govern the acting time by transient control, an electric path (this is called secondary path or secondary coil) across the magnetic path is sometimes added.

The main functions of action element are the selective function for attribute of controlling energy and the time function for governing the time of action.

2.2. The attributes of controlling energy.

The attribute of controlling energy are of the following 4 kinds:

(i) Kind
(ii) Quantity
(iii) Phase
(iv) Frequency

2.3. Time function.

Relays do not necessarily act, due to their selective function, when they receive controlling energy; and after they are actuated, they do not necessarily return to original position even when the controlling energy has disappeared. The process in which a relay operates when controlling energy is received and releases when the energy has disappeared is called fundamental time-action form, and this form will be considered in this section, letting non-fundamental form to be taken up later in the article.

When a relay operates or releases, it is of course apparent that a certain time has to be passed; however, without considering minutely regarding the absolute value of this time, the time function can be classified roughly into the following four forms:

(i) Fast operation—fast release
(ii) Fast operation—slow release
(iii) Slow operation—fast release
(iv) Slow operation—slow release.

These four forms made up of combining fast and slow actions can be considered sufficient in composing ordinary relay circuit, and this is because it is quite sufficient to take up the time relationship between actions of two relays. In short, this classification is qualitative and depends upon purposes.

III. ON CONTACT POINTS.

Contact points, as defined in section 1.1, are composite elements of controlled transmission path, governing at their contact points the transmission of controlled energy. The following is the consideration regarding mechanical contact points of electromagnetic relays.

3.1. The fundamental and sub-fundamental forms of contact points.

The real fundamental forms of contact points are two kinds, opening and closing; but usually an action element has many groups of contact points which are made up of simple forms based on these fundamental forms and also of complex forms composed of various combinations of these simple forms. These simple forms are called here sub-fundamental forms.
Regarding contact elements, mechanical contact points are made of movable and stationary contact elements, sub-fundamental contact points are of one movable contact element and two or more stationary contact elements, and by the motion of movable contact element, without including the connection of path elements, the mutual connection of each of contact elements themselves is made or broken.

Listing in figures the fundamental and sub-fundamental forms, as shown in Fig. 1, circular points and short lines represent movable contact elements, and triangular points show stationary contact elements. The figures also show the released state of relays. In case of operation the moving direction of movable contact element, if it is fixed, has an advantage of indicating whether the relay is in operated state or released state in the circuit; however, on the other hand, it has disadvantage of losing the simplicity of figures of energy transmission paths. For this reason fixed moving direction will not be stressed in this article. Although the Fig. 1 shows only ten types of sub-fundamental forms, number of types is inevitably limited due to mechanical construction of the electromagnetic relays. The designation of types is derived from English expression of actions, taking the initial letter of each word.

Besides above mentioned types, there are other types of sub-fundamental form; such as rotary switch, or sequence switch, etc., which is one of the kinds of electro-magnetic relay, having one or two contact elements which together with other contact elements gradually close or open in a definite direction.

3.2. Some of definitions regarding contact points.

i. Family

The group contact points that are in charge of same action element are classified in same family. The contact points in charge of different ac..on elements are in different family with one another.

ii. "x" contact points

In contact points in the same family, a particular one that operates before other p ints is called "x" contact point. Consequently, "x" contact point usually releases later than other points.

iii. "y" contact points

In contact points in the same family, by special construction, one point that operates later than other points are called "y" contact point. Consequently, "y" contact point usually releases before the other points.

iv. Equal contact points

Contact points of the same type are called equal contact points.

v. Inverse contact points

If acting functions of contact points are opposite from one another, they are called mutually inverse contact points. For instance, in Fig. 1, M and B contact points, DM and DB, $B.DM$ and $DB.M$, and $B.MbB$ and $MbB.M$ contact points are mutually inverse contact points. Of the inverse contact points, those that have two contact elements are called simple inverse contact points, and those having more than two contact elements complex inverse

DESIGNATION OF TYPES	FIGURES OF CONTACT POINT	NUMBERS OF CONTACT ELEMENT
M		2
B		2
BbM		3
MbB		3
PM		3
DM		3
DB		3
B.DM		4
DB.M		4
B.MbB		4
MbB.M		4
MbB.DM		4

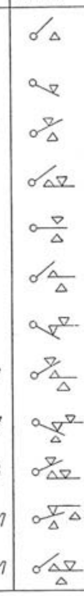

Fig. 1.

contact points. Regarding the simple inverse contact points, there exists a simple relationship as a nature of partial paths which will be discussed later.

vi. Equivalent contact points

In contact point or points, by suitably connecting contact elements with simple path elements when the acting function of these points become equal, these contact points or points are said to be equivalent to one another. As stated in section 3.1, the true fundamental forms of contact points are M and B, and the sub-fundamental f. rms as shown in Fig. 1 are each equivalent to suitable connections of M and B with path elements. These are shown in Fig. 2. In this figure, those listed below $B.DM$ are not given, but they

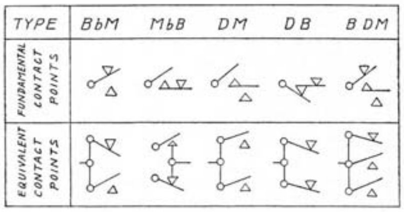

Fig. 2.

are all the same. The presence of such equivalent contact points makes the applications of relays more flexible and economizes the number of contact elements.

3.3. Some theorems regarding contact points.

From the forgoing ideas and explanations, the following few theorems may be considered:

Theorem 1. The operating function of one of the mutually inverse contact points is equal to the releasing function of the other.

Theorem 2. Contact point or points may be replaced by equivalent contact point or points composed of fundamental contact points and simple path elements.

Theorem 3. Mutually inverse contact point or points possess equivalent contact point or points made up of their corresponding simple inverse contact points and simple path elements.

Theorem 4. The equivalent contact points of mutually inverse contact points are inverse with each other.

IV. CONSIDERATIONS REGARDING SIMPLE PARTIAL PATH.

4.1. Meaning and definition of partial path.

In composing relay circuit, it is convenient to discuss dynamic geometrical nature of a part of the circuit, taking up only contact points and simple path elements, and disregarding for the time being the energy source and action elements among composite elements of energy transmission path. The reason for this is that this part is very important in determining the resultant function of many various phenomena. This is called partial path, and is defined as "Partial path is a part of energy transmission path composed of contact points and simple path elements."

When the transmission path of the partial path under consideration is one, it is called simple partial path; and when two or more, it is called complex partial path. While contact points included in a simple partial path are only fundamental contact points, one contact point at least in complex partial path must include three or more contact elements.

The contact points within the partial path are mainly an assembly of contact points of different families, and consequently it is necessary to consider the order of acting time of contact points themselves. This order of acting time is called timing order, and to show the relative priority, it is called pre-timing order and post-timing order. Also the order of geometric positions of contact points in the partial path is termed position order, and again the result of action of contact points affecting the function of partial path is called effect.

4.2. Nature of simple partial path.

Simple partial paths possess very simple and clear characteristics, as contact points within the path are all fundamental contact points.

What will be given in the following

section is the investigation of dynamic geometrical nature of simple partial path. The dynamic geometrical nature of simple partial path, seen from the both ends of the path, means the condition of time expenditure during the process of making or breaking the path of energy. That is, it is the function of process in which a sort of driving-point impedance, taken at both ends of the path, reveals in time sequence two opposite states, zero or infinity, due to the mutual action of contact points when they in the simple partial path operate or release according to a definite order.

Moreover, in this section it is assumed in regard to the time order of action of contact points that (A) it is limited in recurrences of only operation on or of only release in their series of actions, and that (B) it is the same for both cases of operation and release.

Assuming in this way it is possible to see many beautiful and systematic characteristics of simple partial path. When the time order of action follows these assumptions, it is said to be regular.

i. Series connection of contact points.

When contact points or contact points have all of their position order different, they are said to be connected in series; and of these, when every contact point has different position order at each respective point, they are connected in simple series; otherwise they are in complex series (Fig. 3 (*a*), (*b*)).

ii. Parallel connection of contact points.

When contact points or contact points have all of their position order the same, they are said to be connected in parallel; and of these, when every contact point is of the same position order, they are

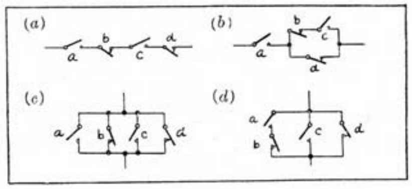

Fig. 3.

connected in simple parallel, and otherwise they are in complex parallel (Fig. 3, (*c*), (*d*)).

iii. Simple partial paths in inverse relation.

When all of the contact points of a simple partial path are replaced by their respective inverse contact points, the resultant simple partial path is said to be in contact-inverse relation with the original partial path. Regarding this from the Theorem 1, the following general theorem may be derived:

Theorem 5. The operating function of simple partial path is equal to the release function of simple partial path in contact-inverse relation. The acting functions of two simple partial paths having contact-inverse relation is called cross-equal.

This shows that it is sufficient to discuss only on either one of the two simple partial paths when expressing the acting function.

Next, taking contact point or contact points of each position order of simple partial path, changing connection of series simple partial path into parallel simple partial path or vice versa is called connection-inverse. In the Fig. 4 (*b*) and (*c*) respectively represent contact-inverse and connection-inverse of a simple partial path of (*a*).

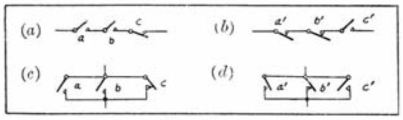

Fig. 4.

iv. Simple partial paths in double inverse relation.

A simple partial path and another simple partial path converted from the former by both contact-inverse and connection-inverse treatment are said to be mutually in double-inverse relation. In Fig. 4, (*d*) is double inverse of (*a*). The contact-inverse, connection-inverse, and double-inverse have the following relationships:

Theorem 6. With respect to one simple partial path, a simple partial path in contact-inverse relation and another in connection-inverse relation are mutually in double-inverse relation.

Theorem 7. With respect to one simple partial path, a simple partial path in contact-inverse relation and another in double-inverse relation are mutually in connection-inverse relation.

Theorem 8. With respect to one simple partial path, a simple partial path in connection-inverse relation and another in double-inverse relation are mutually in contact-inverse relation.

These relations may be easily understood by looking at Figs. 4 and 5. Fig. 5 shows these three transformations in three-dimensional relations, and each of vertices, A, B, C, D, of tetrahedron, is a simple partial path, and also shows that one of them treated by any one of the above three transformations corresponds to one of the other three. Thus each edge, as shown in the figure, means treatment or relationship of three transformations. This is the fundamental principle of the tetrahedron showing the contact-inverse, connection-inverse, and double-inverse relations. From Fig. 5, if two transformations are carried out one after the other, there remains relationship of the remaining one, and if three of them are all carried out one after another or the same transformations is carried out twice, they return to the original form.

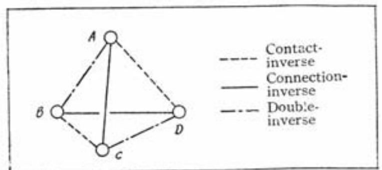

Fig. 5.

Theorem 9. The acting functions of simple partial paths in double inverse relationship are conjugate with each other.

Here, this conjugate function means that when one of simple partial paths is closed or opened, the other is respectively opened or closed. Fig. 6 shows this in its simple condition. The right side of the figure indicates the time characteristic, and now assuming a, a' as being in pre-timing order with respect to b, b', T_{1A}, T_{1B} indicate their times of operation, T_{2A} and T_{2B} their releasing time, solid lines indicate closed state, and the white spaces the opened

Fig. 6.

state. In the figure the lines of one side fill the white space of the other side, and the solid lines themselves do not come together. This in fact is the pictorial indication of conjugate function.

The Theorem 9 is very important in composing circuit, and if a circuit acting in conjugate with any circuit is desired the Theorem 9 must be generally applied.

This idea of double inversion is similar to the idea of inverse network regarding driving-point impedance of so-called transmission network. For instance, taking Fig. 1, page 262, of B. S. T. J., April, 1924, which shows corresponding characteristics of inverse networks, as discussed on Reactance Theorem by R. M. Foster, if inductance and capacity are respectively replaced by B and M contact points, a simple partial paths having double inverse relationship are obtained. These are shown in Fig. 7 in which, however, corresponding inverse contact points are of the same family, that is, their timing-order are not changed, and they are indicated by alphabets with or without prime signs. I and II are corresponding groups of double-inverse simple partial paths, and by combining them further, more complex forms of simple partial paths in double inverse relations may be considered.

Let us now consider the function of mutual action of simple partial paths which are in connection-inverse relation discussed in paragraph iii. From Theorem 5, 6 and 9 the following theorem in regard to the acting functions of to partial paths in connection-inverse relation will be given.

Theorem 10. The acting functions of simple partial path in mutually connection-inverse relation are mutually cross-conjugate.

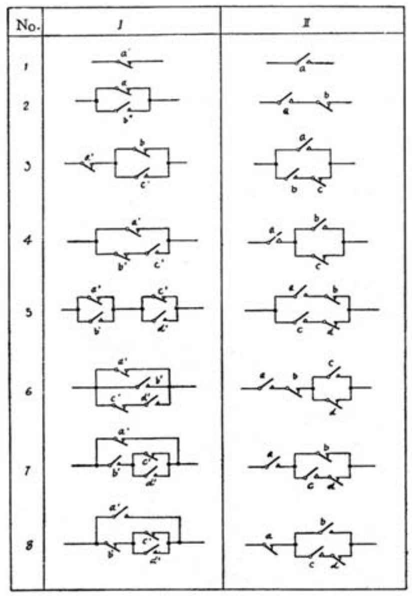

Fig. 7.

Cross-conjugate means that the operating functions of one side are conjugate with the releasing function of the other side. The idea of cross is same as that in the Theorem 5.

Thus, by the Theorems 5, 6, 7, 8, 9, 10, if any one of the acting functions of four simple partial paths related by the principle of tetrahedron is known, the rest will be deduced from it.

Now considering connection of simple partial paths in double-inverce relation from the various examples shown in Fig. 7, another theorem will be given.

Theorem 11. Two pairs of simple partial paths of double-inverse relation being given, if two sets of partial paths, one from each pair, are connected inversely, the resultant pair are also mutually in double inverse relation.

For instance, in Fig. 7, I-♯1 connected in series with I-♯2, and II-♯1 connected in parallel with II-♯2 compose a pair of ♯3.

v. Effect of Contact points

In relay circuits it is often necessary to know which contact point gives the most efficient effect, paying special attention to the function of simple partial path as a whole. In other words, in composing a circuit it is very necessary to determine how to arrange and what types of contact points to be selected in order to avoid the other contact points to act. This effect is also one of acting functions of simple partial path, and consequently, the foregoing Theorem 5 and Theorem 9 can be applied. If the effect is emphasized at all times during actions, the timing order of the contact points becomes a problem, but when it is limited only to the case of finished state of actions of contact points, the timing order does not affect, and it is determined by the types of contact points and the geometric figures of partial paths.

vi. Interchanging position order of contact points.

Generally, in a simple partial path, even when arbitrary two position orders of contact point or contact points that are in different position order are interchanged, there is no change in the acting function of the simple partial path. Consequently, in a simple partial path, its acting function is not related to the position order of contact points. This may be termed as <u>interchangeability of position orders of contact-points</u> in simple partial path.

Theorem 12. Position orders of contact points in a simple partial path are interchangeable.

vii. Interchanging timing order of contact points.

In cases where timing order alone is interchanged, keeping the types of contact points the same, there arises, in general, a change in acting functions; that is, in general, the interchange of timing order of contact points is impossible.

viii. Nature of simple partial path composed of equal contact points.

Such a simple partial path has its contact points made up entirely of M contact points or B contact points; and as these are mutually in contact-inverse relation, it is sufficient to express, from Theorem 5, the nature of the path with only one of the two kinds of contact points. Consequently, that which is made of only M contact points will be considered.

Theorem 13. The operating and releasing effects of a simple partial path, with M contact points connected in simple series, are respectively possessed by contact points of the initial and final timing order.

Theorem 14. The operating and releasing effects of a simple partial path, with M contact points connected in simple parallel, are respectively possessed by contact points of the initial and final timing order.

As Theorems 13 and 14 are related with simple partial paths of mutually contact-inverse, it is clear from Theorem 10 that one is able to lead from the other. Moreover in such simple partial paths, even when timing order is interchanged, the acting functions do not change. That is, the following theorem will be derived.

Theorem 15. In case of a simple partial

path composed of equal contact points connected in simple series and simple parallel, it is possible to interchange the timing order.

ix. Nature of simple partial path composed of two contact points.

When two contact points composing a simple partial path are equal, the foregoing general principles are of course applicable. Therefore, cases where there are two inverse contact points will be considered at this time; for such plain simple partial path, the following theorem is obtained:

Theorem 16. In case of a simple partial path composed of two inverse contact points, interchange of timing order and transformation of contact-inverse change its acting functions equally.

4.3. Analytical expression of acting functions.

i. Acting impedance of contact points.

Contact points included in simple partial path are of fundamental contact points of M and B, and path elements are simple path elements. Therefore, looking at their acting functions from the standpoint of the idea of driving-point impedance, the acting impedance becomes discontinuously either zero or infinite as time passes on. The function to express such time-action characteristics can be treated, for instance, by a form of Fourier's Integral. It is agreed, however, that the infinity is expressed by letter π for the time being. This agreement is believed to be appropriate when it is considered as follows: The quantities treated here are only two kinds, zero and infinity, and do not contain any other intermediates between them; and zero, one of the two quantities, can be expressed by one of the values of Fourier's integral itself. Therefore, if the infinity is considered as a value different from zero, it may be expressed qualitatively by π, the other value of Fourier's integral.

The acting function of M and B contact points, as shown in Fig. 8, can be expressed by the following equations; where the function $f(t)$ shows the operating function, and M and B are placed to show the type of contact points:

$$f_M(t)=\frac{\pi}{2}+\int_0^{\infty}\frac{\sin(t_1-t)x}{x}dx=\pi \quad t<t_1$$
$$=0 \quad t>t_1 \qquad \cdots\cdots(1)$$

$$f_B(t)=\frac{\pi}{2}+\int_0^{\infty}\frac{\sin(t-t_2)x}{x}dx=0 \quad t<t_2$$
$$=\pi \quad t>t_2 \qquad \cdots\cdots(2)$$

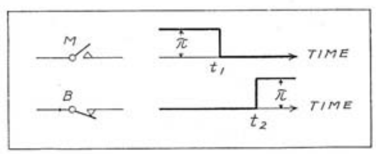

Fig. 8.

Equation (1) and (2) are respectively called the operating impedance of M and B contact points.

Next, expressing the releasing impedance by function $g(t)$, from the Theorem 5 of the cross-equal, the following relations are formed:

$$\left.\begin{array}{l} g_M(t)=f_B(t) \\ g_B(t)=f_M(t) \end{array}\right\} \qquad \cdots\cdots(3)$$

As stated above, the contact point impedance produces two opposite values alternately and discontinuously with time, and compared with the fact that impedance of the pure-reactance network in transmission circuits changes continuously with frequency and produces alternately resonance and anti-resonance at special frequencies, it is surprising to note the singular difference and contrast between the two.

ii. Acting impedance of simple partial path connected in simple series.

Here it is assumed that actions of contact points are regular in time. In that case when operating function of simple partial path is expressed by $F(t)$,

$$F(t)=\sum_{p} f_{Mp}(t,t_p)+\sum_{q} f_{Bq}(t,t_q) \cdots\cdots (4)$$

similarly, expressing releasing function by $G(t)$,

$$G(t)=\sum_{p} g_{Mp}(t,t_p)+\sum_{q} g_{Bq}(t,t_q) \cdots\cdots (5)$$

A simple case will now be explained with Fig. 9. In the figure, for the simple partial path (a), assuming B contact point to be in pre-timing order,

$$F(t)=\frac{\pi}{2}+\int_0^\infty \frac{\sin(t-t_1)x}{x}dx+\frac{\pi}{2}+\int_0^\infty \frac{\sin(t_2-t)x}{x}dx$$

$$=(\pi)^{t_1}+(2\cdot\pi)_{t_1}^{t_2}+(\pi)_{t_2}\equiv\pi \cdots\cdots (6)$$

$$G(t)=\frac{\pi}{2}+\int_0^\infty \frac{\sin(t_1-t)x}{x}dx+\frac{\pi}{2}+\int_0^\infty \frac{\sin(t-t_2)x}{x}dx$$

$$=(\pi)^{t_1}+(2\cdot0)_{t_1}^{t_2}+(\pi)_{t_2}$$

$$\equiv(\pi)_{t_1}+(0)_{t_1}^{t_2}+(\pi)_{t_2} \cdots\cdots\cdots (7)$$

Equations (6) and (7) show respectively the impedance shown in (b) and (c) in Fig. 9. The letters indicating time placed on upper and lower right side of ( ) means

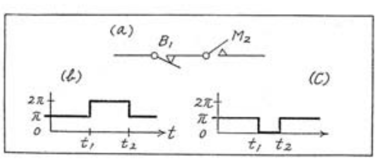

Fig. 9.

that during the time between them the characteristics of ( ) are revealed, and the numbers on the left of π or 0 merely show the number of contact points which produce this condition, and the action impedance of partial path itself is π or 0.

From the above explanation, we get the following theorem:

Theorem 17. Acting impedance of simple partial path connected in simple series, is equal to the total sum of operating impedances or to the total sum of releasing impedances of each of contact points.

What would be the result of interchanging the timing order in Fig. 9? It is to interchange from t_1 to t_2 and from t_2 to t_1 in equation (6). This clearly coincides with the equation (7). Similarly, equation (7) may be changed to equation (6). This is no other than analytical proof of Theorem (16) previously stated.

And in a simple partial path with equal contact points connected in series the part regarding timing order is expressed by

$$\sum_{p}\int_0^\infty \frac{\sin(t_p-t)x}{x}dx \quad \text{or}$$

$$\sum_{q}\int_0^\infty \frac{\sin(t-t_q)x}{x}dx$$

but though the timing order of two arbitrary equal contact points are changed, $\sum_{p}$ or $\sum_{q}$ does not change, and consequently the function of the simple partial path does not change. This is exactly the analytical proof of Theorem 15. Also Theorem 13 regarding effect of contact points of simple partial path with equal contact points connected in series can be proved.

iii. Action impedance of simple partial path connected in simple parallel.

In this case also, as in the case of

transmission circuit, the following theorem is made.

Theorem 18. Action impedance of simple partial path, connected in simple parallel, is equal to reciprocal of the total sum of reciprocal of each operating impedance or of each releasing impedance.

This may be shown in the following equations. But in calculating the addition ought to be done by parts in the order of time sequence.

$$F(t)=\frac{1}{\sum_p \frac{1}{f_{Mp}(t,t_p)}+\sum_q \frac{1}{f_{Bq}(t,t_q)}} \quad \cdots(8)$$

$$G(t)=\frac{1}{\sum_p \frac{1}{g_{Mp}(t,t_p)}+\sum_q \frac{1}{g_{Bq}(t,t_q)}} \quad \cdots(9)$$

For example, considering the simple partial path as shown in Fig. 10, and putting equation (1) and (2) in (8) and (9) respectively, the calculation will be

$$F(t)=(0)^{t_1}+(0)_{t_1}^{t_2}+(0)_{t_2}\equiv 0 \quad \cdots\cdots(10)$$

$$G(t)=(0)^{t_1}+\left(\frac{1}{2}\cdot\pi\right)_{t_1}^{t_2}+(0)_{t_2}$$

$$\equiv(0)^{t_1}+(\pi)_{t_1}^{t_2}+(0)_{t_2} \quad \cdots\cdots\cdots(11)$$

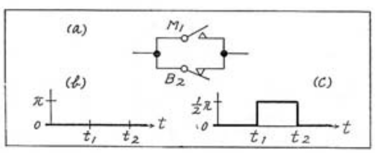

Fig. 10.

If (10) and (11) are compared respectively with the previously obtained (6) and (7), Fig. 9 (a) and Fig. 10 (a), are in their functions, found to be conjugate, but this is evident from Theorem 9 because the two simple partial paths are in double inverse relationship.

iv. Action impedance of simple partial paths in general.

The general simple partial paths are, in fact, the combinations of above-mentioned simple series and simple parallel connections, and in regard to the action impedance as in the case of ordinary transmission circuit theory, series and parallel connections are considered in their order of arrangement.

4.4. The purpose of simple partial path.

A simple partial path is used either as it is or as a part of a complex partial path, but emphasizing on its action form, it may be divided into the following two kinds:—

A. That regards action form at transient time as its main purpose.

B. That regards action form in steady state as its main purpose.

In case of A, the number of opening and closing of the path during the period from the beginning of action until reaching steady state and the time duration are mainly utilized; and it is necessary to make the dynamic geometrical and analytical study as previously mentioned.

In case of B, it is simply for the use of opening or closing of a partial path when series of actions reach the steady state; and analytical study regarding time is not necessary, as it can be easily judged by knowing the type of contact points and their connection figure. However, in this case, it is often necessary to make the acting functions of partial paths, which are formed by contact points of a given group of relays, different from one another, corresponding to various combinations of relay actions. In such casess mathematical treatment of combination is necessary. In general, the number P of such simple partial paths, having its object in closing just one set of simple partial path at the same time by using n number of relays, will be expressed by

$$P=\sum_{0}^{n} rnC_r$$

However, this is true only in case where number of contact points of the same family is assumed to have no limit. If it is limited to be less than P, simple partial paths will be realized only in that number.

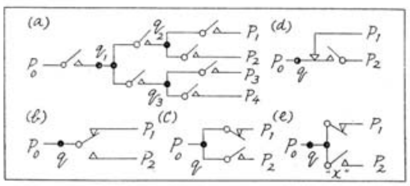

Fig. 11.

V. CONSIDERATIONS REGARDING COMPLEX PARTIAL PATH.

5.1. The meaning and definition of complex partial path.

Complex partial path is in a word aggregation of simple partial paths. Consequently, regarding each of the simple partial paths that compose complex partial path, it is quite reasonable to have its nature as stated previously. However, in the complex partial path, from its purpose, the idea of transfer or selective catch of energy transmission path is necessary.

Looked from the geometrical figures, the complex partial path has a nature of divergence or convergence of energy transmission paths, and exist within it at least one point at which at least 3 paths come together. The simple partial path between such gathering points or between the terminal of path under consideration and its nearest gathering point is termed branch path, and among a group of branch paths that which is closer to the origin of divergence is said to be in higher order of those that are farther away. For instance, in Fig. 11 (a), p_0q_1, q_1q_2, q_2p_1 are each a branch path, and q_1q_2 is in the higher order of q_2p_1. In the most commonly used contact point BbM, as it is clearly shown in Fig. 2, the gathering point ought to be taken at the side of movable contact element. Rotary switch is also similarly considered. In the contact point MbB of Fig. 11 (e), q is to be taken as the gathering point.

5.2. Nature and type of complex partial path.

As complex partial path is a composite body of various simple partial paths, the above mentioned natures of simple partial paths, of course, exist in each branch path. A group of branch paths taken in series may be considered as one of simple partial paths, and the same nature will also exist. Therefore, in discussing the nature of complex partial path, only those which differ from that of simple partial path will be considered. Although there are many types of complex partial path, the discussion will be limited to only those which are most practically used.

i. Effect of contact points in branch path.

Regarding the action of complex partial path, as it takes a form of convergence or divergence, the branch paths of higher order control those of lower order; that is, the action effect is possessed by those of higher order. This may be clearly seen by looking at the following example:

ii. Complex partial path composed of

only M contact points.

This is shown in Fig. 12. The operation progresses from the branch paths of higher order to those of lower order, depending upon which M contact points of the branch paths is closed. Therefore, this is used, for instance, for trunk connection of Relay Automatic Exchange System. Also M contact point has coördinate function.

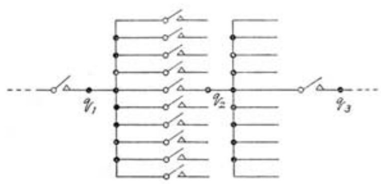

Fig. 12.

iii. Complex partial path composed only of BbM contact points.

M contact point can determine, by its action, only one branch path, but BbM is able to select two branch paths. Such complex partial path has generally the following two classes:

(A) BbM contact points connected in series.

(B) (A) connected in parallel.

In (A) there are, besides, one in which an arbitrary contact point trys to take up the objective path when it operates, and another in which several contact points from higher order to the next operate gradually to take up the objective paths; and they form respectively as shown in Fig. 13 (a), (b). These are applied in, for instance, counting relay circuits.

Class (B) performs shifting of complex partial path of class (A), or select the objective path by combination of operating relays among a group of relays. As an

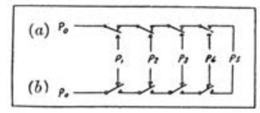

Fig. 13.

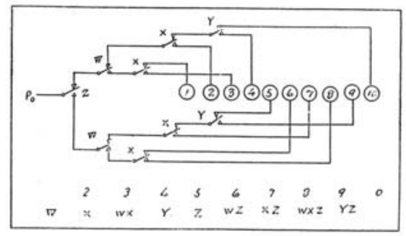

Fig. 14.

illustration the display circuit of call indicator system, is shown above (Fig. 14).

iv. Complex partial path containing M contact points and BbM contact points.

This is the general method which is used in selecting an objective path by the various kinds of combinations of actions of relays. That is, it is used for purposes such as code translation. As an illustration of combining group-selecting function of BbM and coördinate function of M, there is method of "Fifties" and "Fives" marking in Relay Automatic Exchange System.

v. Complex partial path composed of $R.BbM$ contact points.

This is a complex partial path containing contact point forms of electromagnetic relays like rotary switch and selector, for instance; and presents a typical divergent type, with its branch path of higher order connected to movable contact point elements (wiper) and of lower order connected to numerous stationary contact point elements (bank contact points). A part of this is shown in Fig. 15. By connecting these in series an infinite divergence could be obtained, and the path of higher order displays the group-selecting function for those of lower order. This is exactly the theory of step-by-step connection of automatic telephone exchange system. Considering one of these path orders, its acting function is equivalent to the operate-release process of each of M contact points in Fig. 15 (b); and again, considering one of paths of next order in (b), it is equivalent to operation of desired M contact point only. The latter shows the equivalence of relay system with step-by-step system. Moreover, the uni-directional progress of rotary switch is equivalent to (a) and (b) of Fig. 13. This together with the previous case is the cause to produce equivalence between relay system and rotary switch system as in the case of calculating method. From the foregoing discussion the following theorem is obtained:

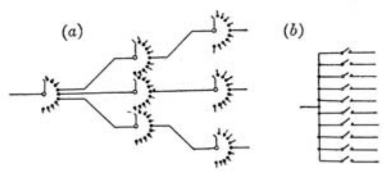

Fig. 15.

Theorem 19. Complex partial path composed of $R.BbM$ contact points, that composed of suitable connection of M contact points, and that composed of BbM contact points are mutually replaceable.

vi. Symmetrical connection of complex partial paths.

Here the symmetrical connection of complex partial paths is a term used when two sets of complex partial paths are connected at their converging sides or at their diverging sides. As combined function of the two sets, it is evident that the former has diverging and the latter converging nature. Illustration of this in case of rotary switch, for instance, is shown in Fig. 16. In case of (a) the total number of simple partial paths that can be formed is $10\times10=100$, and this shows the principle of connecting line-finder to selector. (b) as a whole can be regarded as a simple partial path; and if corresponding contact elements at diverging side are connected, it shows the principle of regenerating method of impulses; and if otherwise, it shows the principle of impulse transformation. The latter is shown in figure (b), and is one example of characteristic 7 of relay circuits, the capability of converting forms of phenomena.

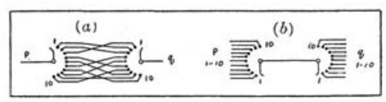

Fig. 16.

VI. CONSIDERATION REGARDING ENERGY TRANSMITTING PATH.

6.1. Introduction.

In IV and V, when functions of partial paths were considered, the idea regarding actions of action elements was touched only implicitly, but effect of actions of partial paths upon action elements was left out of discussion. In this chapter the function of energy transmitting path with action elements and energy source explicitly added to partial path will be considered. For convenience, the energy source is assumed to be direct current.

Before going into the discussion, the idea and definitions of some of the terms will be made clear.

i. When action elements of two or more relays are connected in series, these relays are said to be in relation of action elements-in-series.

ii. When action element of two or more relays are connected in parallel, these relays are said to be in relation of action elements-in-parallel.

iii. When, contact points of some relays are contained in the energy transmitting path having action elements of other relays, the latter with respect to the former is said to be in relation of relays-in-series.

iv. Sub-dividing iii, when an energy transmitting path having an action element of one relay contains only one contact point of another relay, the former is said to be in simple series with the latter. Otherwise, they are said to be in complex series; and according to their types of contact points they are respectively termed *M*-series and *B*-series.

v. When two relays are mutually in series relationship, they are called mutual series, and when contact points of a relay is included in the energy transmission path containing its own action elements, it is called self-series.

These above cases are shown in Fig. 17. The dotted lines are arbitrary partial paths not containing contact points of relays under consideration, and the figure shows the simple partial path under consideration. Also only those relays that show the fundamental time-action forms (see section 2, 3) will be given. In the figure, (*a*) shows action elements-in-series; (*b*) action elements-in-parallel. (*c*), (*d*), ... etc. show various cases of relays-in-series. (*c*) shows complex-*M*-series, (*d*) shows simple series; *B* in respect to *A* is simple-*M*-series; *C* in respect to *A* is simple-*B*-series. (*e*) shows mutual series, in which *A* and *B* are in *MM*-mutual series, and *A* and *C* in *MB*-mutual series. (*f*) indicates self-series, in which *A* is *M*-self-series, and *B* is *B*-self series.

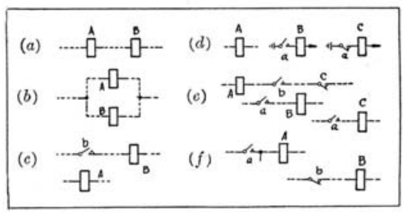

Fig. 17.

vi. A group of relays which is, as shown in Fig. 18, composed by repeating in turn the relationship of relays-in-series are said to be in iterative series. And when they are all in *M* series or all in *B* series, they are called iterative series of same kind, otherwise they are iterative series of different kind.

vii. A relay which is in series with another relay is said to be in rear order with respect to the latter, and the first of the relays in iterative series is called a relay of initial-order.

6.2. Nature of energy transmitting path.

Basing upon the foregoing definitions let us now examine various natures of energy transmission path. In this case, however, the acting functions of simple partial paths shown in dotted lines in Fig. 18, that is, of those not containing contact points of relays under consideration, are left out of discussion for the

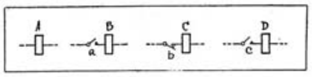

Fig. 18.

time being. Consequently, there will be no question of difference in functions of complex series and simple series. Moreover, it is agreed to advance discussions limiting the time-action characteristics of relays to be of fundamental type.

i. As for a group of relays which are in M-series or in B-series with a certain relay, the time-acting forms of simple partial paths containing their contact points satisfy the condition A of regularity without fail, but not the condition B always. Therefore, the double-inverse relation alone may be discussed, but others may not always be discussed.

ii. The time-action forms of a series of relays in iterative M-series of same kind satisfy both conditions A and B. Consequently, in simple partial paths composed of contact points of these relays possess the beautiful nature explained in Chapter VI (Fig. 19 (a)).

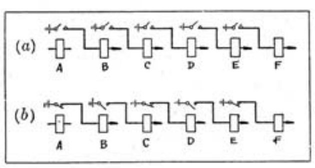

Fig. 19.

iii. In a group of relays in iterative B-series of same kind, any two relays placed one after the other as shown in Fig. 19 (b), are in opposite, operated and released, steady states; and considering the action of the initial-order relay, the action-order of relays of rear-order are the same in case of either its operation or its release. Consequently, in simple partial path made up of contact points of relays of rear order, by replacing contact points of every other relays by inverse contact points, thus making the action order regular, the acting function of this simple partial path is the same as that of the original simple partial path. By converting in this way, it is possible to examine the functions of a simple partial path by various natures of the simple partial path action of which is regular.

iv. The relays which are in MM mutual series, as A and B, in Fig. 17 (e), have the nature of mutually holding their operated condition. This is called the nature of mutual holding of operation. Like A and C, the relays in MB-mutual-series mutually check against the other actions, and have nature of repeating operation-release-operation-release····. This is called the nature of mutual interrupting action.

v. It is clear that A and B, in Fig. 17 (f), are special cases of iv, and have respectively the nature of self holding of operation and self interrupting action.

VII. TIME-ACTION FORMS OF THE RELAY AND THEIR OBJECTS.

7.1. Introduction.

As it has been stated before, the action elements of relays have selective function and time function. In this chapter, considering both of these functions and the nature of energy transmitting path, the action forms of relays which seem to be taken as composite elements of time-action function of relay circuits in general will be discussed by classifying them from the standpoint of their purposes.

The following are their major forms:

i. Forms of fundamental action.
ii. Forms of sustained action.
iii. Forms of interrupting action.
iv. Forms of permanent operation.
v. Forms of permanent release.

7.2. Forms of fundamental action.

These are, as stated in section 2.3, action forms such as to operate when controlling energy is received and to release when the energy is ceased, and have for their object to realize the given phenomena qualitatively. In order to realize given phenomena faithfully and qualitatively, fast-operating and fast-releasing type is to be used, and for this case, like telegraphic relays or impulse relays in automatic exchange system, not only it requires the severe conditions in the time-action characteristics of the relays themselves, but also the characteristics of the energy transmitting path composed of complex path elements become important problems. The slow release action has as its original object to prolong the extinction time of phenomena, but also it is used to connect transiently one phenomenon to the successive phenomena. The slow operating action is as its original purpose to retard the occurence time of phenomena, but it is also used to make one phenomenon to wait for other phenomena.

7.3. Forms of sustained action.

A sustained action means that although the controlling energy is received intermittently, the action of the relay sustains its definite acting condition, and according to forms of action, it is divided into two kinds: Sustained operation and sustained release. The former usually uses slow-releasing type relays, and the latter slow-operating type.

i. Class 1 forms of sustained operation.

This is the form of sustained operation in which a relay in the operated condition after receiving controlling energy sustains the same condition while the controlling energy is received intermittently, for instance, in impulse circuit (Fig. *20*) of Strowger Automatic Exchange system, action of relay B for impulse relay A. This sustaining action shows the function of connecting successive phenonena.

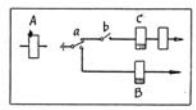

Fig. 20.

ii. Class *2* form of sustained operation.

This is the form of sustained operation in which a relay, in the release condition without receiving controlling energy at first, operates as soon as controlling energy is received intermittently, and during the time of this reception sustains the operated condition. For instance, it is like relay C (Fig. *20*) in impulse receiving circuit of Strowger automatic exchange system. As for relays, fast-operating-slow-releasing type is used. This operation form has the function of replacing a series of successive phenomena by one phenomenon, or has function of recording spaces between groups of phenomena.

iii. Class 3 forms of sustained operation.

This is a form of sustained operation, in which the operation of Class *2* is retarded. Fig. *21* illustrates this form. In the figure, V is vibrating contact point and relay A keeps its magnetic energy at the time V is opened, and obtains its escaping path when V is closed. This type is mainly used, like in voice-frequency relay circuit, to operate direct current relays by alternating current of short period.

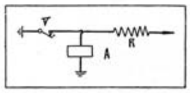

Fig. 21.

iv. Form of sustained release.

For this, relays of slow-operating type are used; for instance, they are used to sustain released condition when controlling energy whose time is shorter than that of operating is received. Consequently, it is best for not leaving the effect of controlling energy that has been received previously. That is, it is best in having long slow-operating time and fast-releasing time.

7.4. Interrupting action form.

Interrupting action form means that although the receiving path of controlling energy is steadily closed, the relay itself keeps the operation-release action and repeats the interruption of transmitting path of controlled energv. The fundamental principle of composition of such relay circuit is to relate the effect of relay operation in such a way as to cut recurrently the receiving path of its own controlling energy, and is one of push-pull action.

i. Class 1 interrupting action form.

This is of B-self series as shown in Fig. 17 (b). In simple relays having this action form as their object there are pendulum relays which convert direct current to alternating current, and in relay circuits there is one like self-release action of rotary switch.

ii. Class 2 interrupting action form.

This form is performed by relays which are in MB-mutual series, as shown by relation of A and C in Fig. 17 (e). It is used as one of impulse generators.

7.5. Permanent action form.

To begin with, the idea of permanent action will be explained. In handling relay circuits in general, depending upon the required time-relation between the given and desired phenomena, relays receiving once the initial controlling energy maintain the operated condition permanently even after the controlling energy has disappeared, unless otherwise regulated; or they maintain the released state permanently even after the controlling energy is received unless otherwise the restraining measure is ceased to be applied. This is the action form that is called permanent action, and the former is termed permanent operation and the latter permanent release.

Ordinarily the permanent operation is called self-holding, and is employed in cases in which it begins its operation due to some phenomena, continues to remain in its operated state even after the phenomena have disappeared, and then releases when other phenomena have occurred. This is exactly the underlying source of characteristic 8 in relay circuit, which is the capability of storing forms of phenomena, and is also a very important reason for its characteristic 7, which is capability of converting forms of phenomena.

Considering methods of operating electromagnetic relays permanently, though there are many kinds, depending upon relays, as far as electromagnetic relays are concerned, there are following four classes:

Class 1. The energy for permanent operation is supplied through other transmission paths than the initial transmission path of the controlling energy.

Class 2. Resisting energy at operating position is negative.

Class 3. Resistive energy of operating position is zero.

Class 4. Mechanically hold at operating position.

Class 1 has a great freedom of controlling transmission of holding energy, though it requires the supply of energy during the operation; and consequently, it has advantage of having in the circuit very broad range of applicability, and is the most commonly used means for recording. Listing the practical means of these forms,

Class 1-A Permanent operation form. This is *M*-self series like Fig. *22* (*a*).

Class 1-B Permanent operation form. It is *MM*-mutual series like Fig. *22* (*b*).

Class 1-C Permanent operation form. It is a form used in a relay having many windings one of which being supplied with enough energy to hold the operated state of the relay when it is once operated, but not enough to operate it with the energy.

Class 2-A Permanent operation form. This is a form used in the permanent polarized relay called Two Position Type. It is shown in Fig. *22* (*d*).

Class 2-B Permanent operation form.

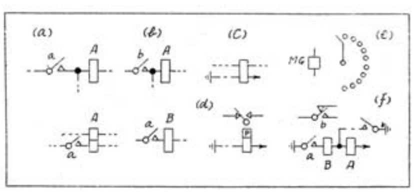

Fig. 22.

This is a case in which the resisting energy by gravity becomes minimum potential in operating position.

Class 3 Permenent operation form. This is a case like rotary switch or sequence switch. If it is considered that the wiper prepares for its motion while the controlling energy is being received and begins to step when the controlling energy disappears, the equivalent relay circuit will be obtained as shown in Fig. *22* (*f*). That is, receiving controlling energy, A operates and closes contact point *a*, but B takes up the Class 1 permanent release form which will be explained in the next paragraph, and does not yet operate; and when the transmitting path of controlling energy is opened, due to *M*-self series of A, B performs with A the permanent operation and operates the contact point *b*. Therefore regarding the controlling energy and the contact point *b* such relay circuit is equivalent to one of contact points of rotary switch. This lays the foundation that a rotary switch and a set of group relays composed of such relays are truly equivalent.

7.6. Permanent release form.

All relays operate selectively corresponding to attributes of controlling energy as already stated in section *2.2*. The fundamental principle for permanent release is to provide suitable checking means in accordance with this selective function. Here will be considered regarding direct current electromagnetic relays the practical means with respect to quantity, phase, and frequency as attributes of controlling energy. They may be divided as follows:

Class 1. The checking means is concerned with quantity.

Class 2. The checking means is concerned with the phase.

Class 3. The checking means is concerned with frequency.

Class 1-A Permanent release form.

This is a form to check the operation by series impedance limiting the value of controlling energy below the operating value; and to remove this checking means, it can be done by suitably decreasing it, but usually it is done by taking it out or by short circuiting, as shown in Fig. 23 (*a*). For series impedance also high resistance relays are sometimes used; in this case it is the usual means, by their presence in series, to release A and to operate B at the same time.

Class 1-B Permanent release form.

This is to limit the controlling energy below operating value by the presence of parallel impedance; and its simplest form is short-circuiting, but usually there accompanies some impedance in series. In Fig. 23 (*b*), A is the relay under consideration, B the parallel impedance, and C the series impedance. In order to remove this checking means, the parallel impedance is taken out. Parallel impedance is sometimes low-resistance relay or short-circuit, and each of these shows the condition of gathering point q and are used for the purpose of not operating A.

Class 1-C Permanent release form.

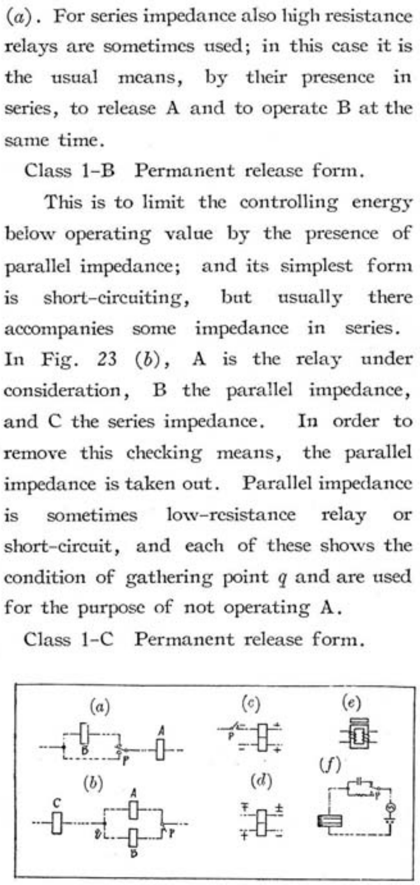

Fig. 23.

This is a form used in relay having many windings, in which the controlling energy is divided into each of the windings, and unless all of the energy is received, it does not operate. This is shown in Fig. 23 (*c*).

Class 2 Permanent release form.

This form is used in relay having two windings; and when one alone receives the controlling energy or when two of them receive currents of opposite direction, it does not operate; and when two windings receive currents of same direction it operates. Such a relay as that having single magnetic path, or so-called shunt-field type, or three-pole electropolarized relay, etc., realizes this form. Fig. 23 (*d*) and (*e*) show this form.

Class 3 Permanent release form.

This form is used when the controlling energy contains both direct and alternating currents, in which alternating current does not operate and direct current alone operates the relay. For instance, as shown in Fig. 23 (*f*), the direct current relays having copper sleave and slug suppresses the direct current by condenser connected in the transmission path of controlling energy. This is used in the ringing circuit of automatic telephone exchange system.

The foregoing discussions are made on permanent operations and permanent release separately, but there are relay circuits having combination of these two forms. For instance in Fig. 22 (*f*), at first during the time while A is receiving the controlling energy, B take the Class 1-B permanent release form; but when the controlling energy is cut off, A and B perform the Class 1-A permanent operation form.

VIII. SOME OF FUNDAMENTAL TYPES OF RELAY CIRCUITS.

Thus far an outline of the most fundamental action forms that underlie in electromagnetic relay circuits is given. This chapter, however, will be devoted to considerations on a relay circuit that will definitely perform a concrete object and on some of the basic fundamentals of its intermediate parts which are of some importance, as to their purposes and method of compositions. Thus by considering such fundamental forms, the composition of relay circuit system which satisfies both given phenomena and desired phenomena as a whole become quite easy.

8.1. Types of counting and recording circuit.

As a method of segregating each of given group phenomena and desired group phenomena in relay circuits, definite numbers or groups of numbers are given to correspond to each of these group phenomena, and these are treated as such numbers or group of numbers in the intermediate stage of circuit. In order to attain these numbers the usual method is to use number of impulses.

In treating such phenomena with their corresponding numbers, three steps are necessary: first, the given phenomena are converted into the corresponding numbers; next, these numbers are counted and recorded; and finally, these numbers are converted back to the desired phenomena. In this chapter only the usual type of counting and recording of intermediate stage will be considered. This is a type of storing forms of phenomena.

When the content of counting and recording is analysed, it is found to be composed of three parts; namely, the main part of counting and recording, a part that assigns the numbers to be recorded to respective groups (count-shifting), and another part that takes up the recording of the main part and retains the recording even after the main part is released (record shifting).

i. The main part of counting and recording.

The elements of counting and recording require first permanent operating form and contact points that correspond to each number. To satisfy such requirements, relay circuits ought to be of stepping mechanism, such as rotary switch or its equivalent, as shown in Fig. 24. In the figure, S is contact point to act according to impulses, and the paths pp_1, pp_2, $\cdots$ represent paths to recorded numbers.

ii. Count-shifting.

If it is desired to record some groups of numbers one after another, the main parts of counting and recording are provided as many as there are groups, and numbers are led into the main parts that correspond to the each group. For this, it

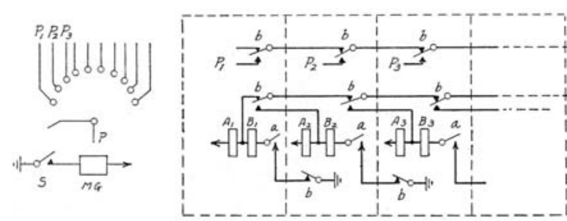

Fig. 24.

is necessary to record number of groups by representing the beginning and ending of numbers of the some group, and the usual method is Class *2* sustained operation form which is to operate while receiving a series of impulses, and to release when receiving ends. Fig. *25* shows one of illustrations. In (*a*) and (*b*), the relay *C* displays the Class *2* sustained operation form. (*a*) is used for count-shifting of many groups of numbers, and (*b*) of only two groups, as in case of a connector.

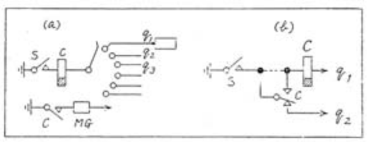

Fig. 25.

iii. Record shifting.

In order to utilize the main part of counting and recording economically, the recording is shifted to other simpler circuits and the main part itself is released to prepare again for the following recording operation. For receiving shifted recording, a permanent operation form is used, but of which the most commonly used form is Class 1-A.

Moreover, as objects of counting and recording, it is possible to record addition, subtraction and multiplication of many numbers.

In the Strowger automatic telephone exchange system, counting and recording parts having stepping mechanism are connected in series to select the desired path; and in the relay automatic telephone exchange system, by using counting circuit of relays and complex partial paths composed of their contact points, the desired path is selected; and also in supervisory system, by assigning corresponding numbers to controlling keys and controlled apparatus, the desired controlled apparatus are selected in the similar manner. In inter-call telephone system and train despatching telephone system also desired telephone is selected in the similar way.

As for types that assign numbers to corresponding phenomena, when the phenomena are few, simply numbers of units or of tenth of units are used; but when the phenomena are many, either they are divided gradually from large to smaller groups, so that their total sum of numbers does not necessarily be a definite number, or they are assigned to number-codes. The former is used in, for instance, step-by-step type automatic exchange system, and the latter, for instance, in *W. E.* type train despatching telephone system.

8.2. Types of generating circuit of assigned numbers.

Let us now consider the form of generating the assigned numbers provided when one of the given phenomena has occurred. Assuming the assigned numbers to be represented by impulse numbers, when the composite substance is analysed, it is made of three parts: generating circuit of impulses, circuit to count and record the impulse current numbers, and circuit to determine the assigned number related to the other circuits.

Impulse generating circuit uses usually either the class *2* interrupting action form which is in *MB*-mutual series or relays that respond to mechanically interrupted currents. The circuit of counting and recording is already stated in the foregoing section; the next consideration is on very important idea of marking which relates determination of assigned numbers to counting and recording and, when recording is finished, makes the transmission of impulses ineffective or discontinued. Marking is to set up different condition on the assigned recording point than on other points, and has object to make ineffective or discontinue the transmission of impulses when the counting and recording has progressed to that point. To make the transmission ineffective, from the idea of contact effect of simple partial path, as stated previously, a suitable form of simple partial paths may be taken. To discontinue the transmission, to the circuit of interrupting action a permanent release form may be added, or cut out the part causing the transmission. Thus, as circuits of generating assigned numbers there exist great many kinds due to the fact that many different ways may be taken regarding their contents. However, the ideas of composition are the same, so that in Fig. 26 only one illustration is given. M contact points is for marking; $L_1 L_2$ forming a simple partial paths are to transmit impulses; INT is mechanical interrupting point; and S is the energy source of this circuit. The relay A responds to marking; B performs releasing of circuit; and C takes charge of impulse current transmission. M contact points may either be that of rotary switch or of relays in case of key sending.

Also in Fig. 26, it is to generate the assigned number itself, but if the starting point of impulse generation is given at the points of the assigned number, it is reasonable to obtain the difference between assigned number and a definite number. This is used, for instance, in revertive

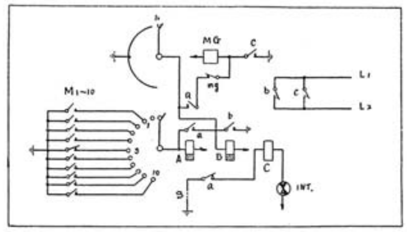

Fig. 26.

impulse control system of outstepping counting relay circuit in rotary type automatic telephone exchange system.

Moreover, the circuit in Fig. 26 is primarily for getting single set of numbers, but if it is necessary to obtain groups of numbers, the method is to shift the marking effects that correspond the groups of numbers in order, so that the transmission of impulses will begin from the numbers of initial group to the following groups until the entire operation is completed.

As it is apparent from the above explanation, the generating circuit of assigned numbers can be utilized as a converting method of phenomena by the principle of marking.

8.3. Types of storing and generating numbers.

When the type of counting and recording stated in section 8.1 is used for the marking in generating circuit of assigned numbers discussed in 8.2, it is able to perform both storing and generating numbers.

In transmitting numbers over a long distance, when impulse repeating is performed many times, the impulse current ratio will gradually be destroyed. In the step-by-step type automatic telephone exchange system, by the use of Class 1 and

Class 2 sustained operation form the safety region of impulse current ratio is determined; consequently, as distortion of the impulse current ratio is undesirable, in such case it is quite reasonable to record impulse number once any way and then to transmit again the impulse of correct ratio. This is what is called the principle of regenerative impulse current transmission.

The correspondence betweem the counted and recorded numbers and the numbers to be generated may be selected at will merely by means of connecting them. (Fig. 16 (*b*).) This is exactly the principle of director-type automatic telephone system.

8.4. Type of self- selection.

As a means of selecting paths in relay circuits, if impulses of a definite corresponding number are transmitted, the type of counting and recording may be used, but this may be considered as a type of externally effected selection. For this there is antoher type in which selection is made only by marking and does not necessarily require the transmission of impulses although numbers are given to the desired paths. Such a type may be called here the self-selection type.

As for the self-selection type, it is only necessary to eliminate impulse transmission in the generating circuits of assigned numbers, given in section 8.2, and it is to utilize the counting and recording of interrupting action and to stop the action at the marked point. For instance, rotary line switch and line finder circuits are of this form. The former is called forward selection and the latter backward selection, taken from their connecting direction, but their principles are the same.

8.5. Type of representing phenomena by codes.

Now another type of representing phenomena will be considered here, that is, the type of representing them by codes.

In order to convert the codes back to each corresponding phenomenon, relays should be provided to correspond the elements of each code, and the corresponding partial path composed of their contact points should display its effect by corresponding combination of the relay actions. An example, in Fig. 14, is shown where, by using 4 relays, 10 phenomena are represented.

To control the actions of relays that represent these code elements, it is only necessary to provide as many energy transmission paths as there are elements; but for long distance there is economical limit to the number of connecting lines, and consequently considering, as attributes of code, the current, its amount, polarity and frequency, there arises necessity of either using combination or so-called distributor type, which means further effective increase of connecting lines must be used.

In the code call indicator, the codes are combinations of small amount of positive current, large amount and small amount of negative current; and in the four frequency signal system the codes are combinations of 4 frequencies, 500, 600, 750, and 900 cycles. Both of these assign the numbers from 0 to 9 to partial paths similar to those shown in Fig. 14.

The distributor type is generally employed in teleprinting system but for this instead of partial paths clutch mechanisms are used. In the Dualsystem which employs + and − as elements of codes, when composed by n units, 2^n different combinations are obtained. For telegraph using alphabets, 5 unit type is used and total of $2^5=32$ combinations are secured, and for Japanese telegraph, total of $2^6=64$ combinations can be obtained; however, in the former 31 combinations are used, and for the latter 63 combinations are employed.

If Dreiersystem, which uses +, − and 0 as 3 elements of codes, is composed by n units, in general 3^n different combinations are to be obtained.

8.6. Type of preventing double connections of sectional circuits.

In complex relay circuits unit sectional circuits that perform one function in each section are often connected in series, such as in automatic telephone system. In such cases there are usually a number of unit sectional circuits which function similarly in each stage, consequently, the one in use ought to be protected from being doubly connected by another. For this reason, at each connecting point between two stage a testing circuits is inserted to see if the latter of the two is in use. To distinguish whether the circuit is in use or not, different conditions are made at the connecting point; so that, for instance, the testing relay in the circuit of the former would not operate if the latter is in use. Fig. 27 shows an outline of the testing method. (a) and (b) both illustrate the use of Class 1-B permanent release form, and the testing relay A cannot operate at point T

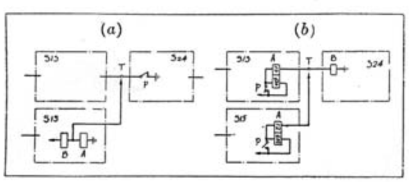

Fig. 27.

while the following section is in use due to the existence of low resistance or shunt in parallel. (a) is the principle used in rotating motion of rotary line switch and in revolving motion of selector in Strowger type automatic telephone system, and (b) in preselector circuit of Siemens type automatic exchange system.

8.7. Type of holding operation of sectional circuits in series.

In case of relay circuit in which group of sectional circuits of various functions are connected in series, in order to hold their operated state the method of shifting Class 1-B permanent operation form of MM-mutual series in orderly sleps to the sections of the following stages as the given phenomena occur. An example is shown in Fig. 28. The figure shows the relationship of connection in Strowger type automatic exchange system. M on one side of MM-mutual series is b contact point, and the figure shows the end of process in which b has been shifted from the selector of initial stage to the selector of the following stage and then finally to the connector.

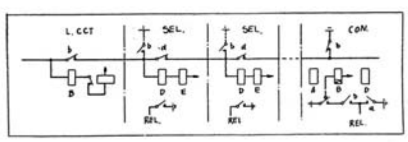

Fig. 28.

IX. CONCLUSION.

In the foregoing Chapters, an outline of somewhat systematic considerations on the fundamentals of composition of relay circuit, especially of electromagnetic relay circuit, has been presented. Heretofore, very little discussion and papers are given on the systematic theory on this phase of relay circuits; therefore, out of sheer necessity, many new words, definitions, and theorems are given. However, this being the first trial, the writer is willing to bear the blame or any criticism for being too dogmatic or too crude. Also regarding acting function of partial paths, much anticipation is to be expected in further investigation. The writer sincerely hopes that the readers will extend him their generous criticisms and instructive comment on the subject presented in the paper.

In conclusion, he also wishes to thank Dr. Y. Niwa and Mr. Y. Shimazu who have given him their utmost encouragement, valuable suggestions, and consultations.

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PART II

This section gives the summaries of articles appearing in the Journal of the I. T. T. E. of Japan published in Tokyo in the Japanese language. The following resumes are extracts from the January, February and March Journals of 1936.

SOME PROPERTIES OF THE GROUP OF SIMPLE PARTIAL PATHS IN THE RELAY CIRCUIT

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This is a detailed discussion on some properties of a group of simple partial paths which are grouped in accordance with the principle of tetrahedron (Refer to Fig. 1) given by the writer in the paper entitled "The Theory of Relay Circuit Composition".[1]

First, by considering the mathematical meaning of the group composition mentioned above, it is made clear that the four kinds of transformations——contact-inverse, connection-inverse, double-inverse, and identical transformations——compose the so-called Abel's group treated in the group theory. Then designating these transformations by letters and expressing the relationships among them mathematically, various physical meanings are shown from them.

Next a method of pictorially representing the acting functions of partial paths is

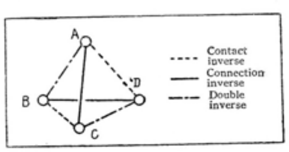

Fig. 1.

proposed, and by presenting some simple figures as examples for the case in which are satisfied the conditions A and B of the regularity of the acting order,[1] the variation in the acting functions due to various transformations are clearly pointed out.

In the previous paper[1] only a brief explanation was given about special methods of connection-inverse and double-inverse transformations, and therefore in this paper the methods of transformations are extended to the most general form. As long as the general method of connection-inverse transformation is made clear, by applying the contact-inverse transformation afterward, the double-inverse transformation can easily be performed. Hence, the general method of the connection-inverse transformation will be given here. This general method is made up of the following two treatments:

(1) Considering for the sake of convenience that two terminals of a given partial path are to be closed, from this circuit select meshes in such a way that each branch path will become the common

coupling element between two and only two adjacent meshes, and designate them by numbers.

(2) Place as many points as there are meshes selected as above and designate them by numbers. Then connect two of these points by the branch path which is the common coupling element between two adjacent meshes, numbers of which corresponding to the numbers of the points, until all of the branch paths are drawn. Then open the branch path not containing contact points, and make the opened ends the two terminals of transformed partial path.

For instance, take Fig. *2* (A) in which a given partial path is shown in solid lines. In order to obtain another partial path which is in connection-inverse relation with this, follow the process as shown in Fig.

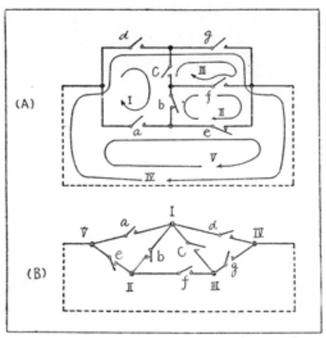

Fig. 2.

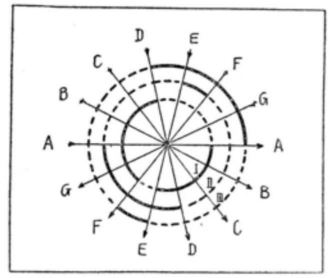

Fig. 3.

2, and the transformed partial path is obtained as shown in solid lines in (B). Each of the partial paths shown in Fig. 7 in the first paper[1] can be regarded as a special case of this transformation.

Fig. 3 represents examples of the acting functions of the simple partial paths of (A) and (B) in Fig. *2* and of a simple partial path in double-inverse relation with (A) respectively by the circular figures I, II, and III.

Finally, for the acting functions of simple partial path after it has undergone the above mentioned general double-inverse transformation, it is analytically proven that Theorem 9,[1] that is, the conjugate nature of the acting functions, does exist.

REFERENCE

(1) A. Nakashima: The Theory of Relay Circuit Composition, Nippon Electrical Communication Engineering, No.3.

THE REJUVENATION OF GRANULAR CARBON IN MICROPHONES

Tamotsu Nishina, Member, **Gen Wachi** and **Toshio Ebihara**, Associates
(The Electrotechnical Laboratory, Ministry of Communications, Japan)

It is a commonly known fact that carbon granules used in telephones possess a certain limited life and that the communicating efficiency gradually decreases as the granules

THE THEORY OF EQUIVALENT TRANSFORMATION OF SIMPLE PARTIAL PATHS IN THE RELAY CIRCUIT

Akira Nakasima, Member

Masao Hanzawa, Member

(Nippon Electric Co., Ltd., Tokyo)

SYNOPSIS

This paper discusses the equivalent transformation of simple partial paths in the relay circuit, which means transformation of connecting forms without changing their acting functions.

A simple partial path is, in general, composed of many various simple partial paths, and the equivalent transformation of such a composite simple partial path is discussed in the following two ways:

(i) To consider each component simple partial path as a unit, without discussing its internal structure.

(ii) To consider the types and acting time of all contact points contained in the composite simple partial path.

In order to develop the theory of equivalent transformation a new method of algebraic treatment is proposed to express goemetrical figures and equivalency of acting functions of simple partial paths, and it is shown that problems of equivalent transformation can be mathematically treated in a very easy manner.

I—1. ALGEBRAIC EXPRESSION OF SIMPLE PARTIAL PATH

A complicated simple partial path is, in general, composed of several component simple partial paths which are connected in various ways. To begin with, the form of algebraic expression for such a composite simple partial path as one made up of series and parallel connection is considered.

Take arbitrary three simple partial paths A, B and C, and connect them in either series or parallel to form a composite simple partial path X or Y respectively. Now the author adopts a rule by which these simple partial paths are to be expressed as following:

In case of series connection,

$$X=A+B+C \quad \cdots\cdots(1)$$

In case of parallel connection,

$$Y=ABC \quad \cdots\cdots(2)$$

When the acting functions of these simple partial paths are considered, it is clearly evident that the associative and commutative laws hold in the right hand sides of equation (1) and (2).

Next it will be shown that when the above mentioned rule of algebraic expression is adopted the distributive law holds true with regard to composite simple partial paths, that is,

$$A(B+C)=AB+AC \quad \cdots\cdots(3)$$

The equal sign means that the acting functions of two composite simple partial paths represented respectively by the left and right hand terms are equal. Consequently, it will be sufficient to prove that the acting functions of these two composite simple partial paths are equal in respect of time and in all combinations of acting impedances

This is a condensed translation of the original paper which appears in Japanese in the Journal of the Institute of Electrical Communication Engineers of Japan, No. 165, December 1936, No. 167, February, 1937.

of three component simple partial paths A, B and C.

As the acting impedance of simple partial path is limited to only two values, zero and infinite, the total number of all possible combinations of acting impedances of these three simple partial paths is $2^3=8$. Therefore, the acting impedances of the composite simple partial paths represented by the left and the right hand terms of equation (3) will be examined in eight combinations as shown in Fig. 1. In the figure, the solid lines and the dotted lines show respectively the zero and the infinite values of acting impedances.

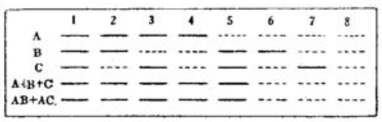

Fig. 1.

It is seen, from Fig. 1, that the acting functions of the two composite simple partial paths $A(B+C)$ and $AB+AC$ are always equal, that is, it is evident that the equation (3) always holds true.

Equation (3) may be graphically shown as in Fig 2.

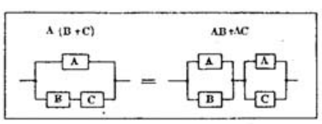

Fig. 2.

From equation (3) we have the following equations:

$$(D+E)(B+C)=(D+E)B+(D+E)C$$
$$=DB+BE+CD+CE \qquad \cdots\cdots(4)$$

$$(A_1+A_2+\cdots\cdots)(B_1+B_2+\cdots\cdots) \qquad \cdots\cdots(5)$$
$$=\sum A_jB_k$$
$$j=1,\ 2,\cdots\cdots$$
$$k=1,\ 2,\cdots\cdots$$

From the proof given above, it is now obvious that, if a composite simple partial path is expressed algebraically by forms of sum and product of its component simple partial paths, the associative, commutative and distributive laws do all hold true as regards the equivalency of acting functions of composite simple partial paths.

In this paper notations are used such that a simple partial path which is in double-inverse relation with simple partial path A is represented by $\bar{A}$ and that a simple partial path always giving infinite impedance or zero impedance is represented respectively by p or s.

I—2. LAW OF ELIMINATION

It is sometimes the case with a composite simple partial path in which elimination of some of its component simple partial paths from it does not make any change in its acting function. This gives a sort of equivalent transformation method. Some of the laws of elimination are given below:

(a) When a composite simple partial path contains several number of same simple partial path A only,

$$nA=A,\quad A^n=A \qquad \cdots\cdots\cdots\cdots (6)$$

where n is any positive integer.

(b) When two component simple partial paths are in double-inverse relation,

$$\left.\begin{aligned} A+\bar{A} &= p \\ (A+\bar{A})+B &= p \\ (A+\bar{A})B &= B \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots (7)$$

$$\left.\begin{aligned} A\bar{A} &= s \\ (A\bar{A})B &= s \\ A\bar{A}+B &= B \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots (8)$$

(c) When a composite simple partial path is composed of two or three kinds of component simple partial paths,

$$AB+A=A \quad \cdots\cdots (9)$$

$$A(A+B)=A \quad \cdots\cdots (10)$$

$$A(B+C)+(A+BC)=A+BC \quad \cdots\cdots (11)$$

$$A(B+C)(A+BC)=A(B+C) \quad \cdots\cdots (12)$$

These laws of elimination are graphically shown in Fig. 3.

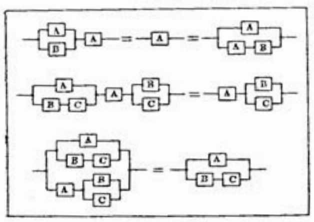

Fig. 3.

I—3. SERIES—PARALLEL TRANSFORMATION

A composite simple partial path having its component simple partial paths connected in parallel can be transformed equivalently into series form by the distributive law mentioned above.

However, the equivalent transformation of a series form into a parallel form is not always possible just by applying the distributive law alone. In such a case, the law of elimination must be applied in conjunction with the distributive law. Some examples are shown below:

$$AB+C=(A+C)(B+C) \quad \cdots\cdots (13)$$

$$AB+CD$$
$$=(A+C)(A+D)(B+C)(B+D) \quad \cdots (14)$$

$$ABC+D$$
$$=(A+D)(B+D)(C+D) \quad \cdots\cdots (15)$$

$$AB+C+D$$
$$=(A+C+D)(B+C+D) \quad \cdots\cdots (16).$$

where A, B, C and D are arbitrary simple partial paths.

These relations are shown in Fig. 4.

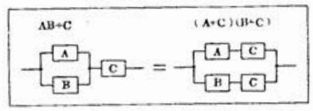

Fig. 4-A.

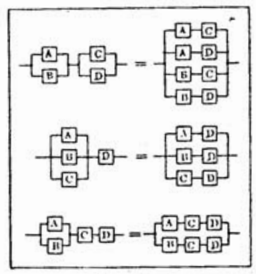

Fig. 4-B.

I—4. EQUIVALENT TRANSFORMATION FROM BRIDGE FORM TO SERIES-PARALLEL FORM

The equivalent transformation of a composite simple partial path composed of five arbitrary simple partial paths connected in bridge form, as shown in Fig. 5, will now be considered.

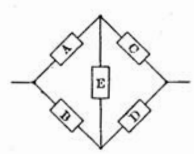

Fig. 5.

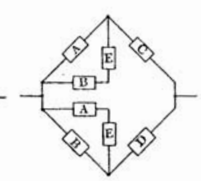

Fig. 6.

A composite simple partial path of such a form as shown in Fig. 6 is first made up of five kinds of component simple partial paths contained in the original composite simple partial path of bridge form and its acting function is compared with that of the original one by resorting to the means indicated in Fig. 1. This comparison shows that the two composite simple partial paths are always equivalent.

In other words, a composite simple partial path of bridge form can be always equivalently transformed into that of series-parallel form.

Therefore the acting impedance of Fig. 5 may be expressed by the following forms

$$\{A(B+E)+C\}\{B(A+E)+D\}$$
$$=AB+CD+ADE+BCE \cdots\cdots\cdots\cdots(17)$$
$$=(A+C)(B+C+E)(B+D)(A+D+E) \cdots\cdots\cdots\cdots\cdots(18)$$

and they may be graphically shown in Fig. 7.

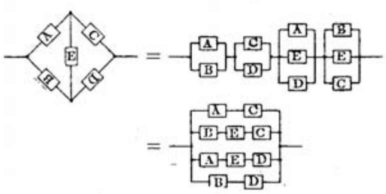

Fig. 7.

I—5. LAW OF PARALLEL SEPARATION

Let us take a case such as shown in Fig. 8 where A and B are any arbitrary simple partial paths and K a three-terminal complex partial path, and consider a method of transforming it into a parallel form.

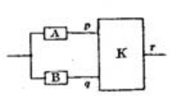

Fig. 8.

Now the transmission circuit theory teaches us that a three-terminal passive network can always be equivalently transformed into a Δ form of three impedances. Therefore the complex partial path K can be also equivalently transformed into a Δ form one composed of three supposed simple partial paths, and Fig. 9 is taken as equivalent to Fig. 8.

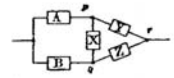

Fig. 9.

Fig. 9 is of the same form as Fig. 5, so that if the same method of transformation from Fig. 5 to Fig. 6 is applied to Fig. 9, the equivalent transformation shown in Fig. 10 will hold true. This is what may be called the law of parallel separation.

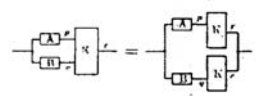

Fig. 10.

This law holds also true even if a complex simple partial path has any number of terminals. Consequently, how much complicated a given simple partial path may be, it can be equivalently transformed by this law into a series-parallel form and be expressed by algebraic expression discussed above.

I—6. EQUIVALENT TRANSFORMATION WHEN SIMPLE PARTIAL PATHS IN DOUBLE-INVERSE RELATION ARE CONTAINED

The equivalent transformation of a composite simple partial path containing component simple partial paths which are mutually in double-inverse relation plays an important part in the algebraical treatment of passive networks in the relay circuit. Some examples of such transformation will be given below for reference.

(i) $A\bar{A}B=s$ ··························(19)

(ii) $AB+\bar{A}B=(A+\bar{A})B=B$ ······(20)

(iii) $A(\bar{A}+B)=AB$ ····················(21)

(iv) $(A+B)(\bar{A}+B)=B$ ··············(22)

(v) $A+\bar{A}B=B+A\bar{B}=A+\bar{B}$ ······(23)

These relationships may be graphically shown in Fig. 11.

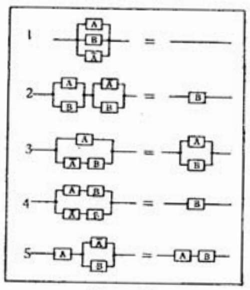

Fig. 11.

Also the equivalent transformations from bridge form into series-parallel form are shown in Fig. 12.

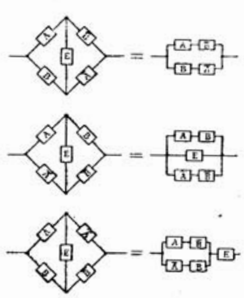

Fig. 12.

II—1. ALGEBRAIC EXPRESSION OF INTERNAL STRUCTURE OF SIMPLE PARTIAL PATH

In Part I, considerations were made on a composite simple partial path with its component simple partial path as units and no special study was made on their internal structure; in other words, the considerations were made rather from the macroscopic point of view. In Part II, however, an extensive discussion of the theory of equivalent transformation of these component simple partial paths regarding their internal structure will be made, that is, from the microscopic stand point.

Now, a simple partial path is made up by combining contact points and simple path elements. In order to express its acting function algebraically, it is quite sufficient to consider just the following four factors which control the dynamical geometric nature of simple partial path.

(i) Types of contact points

(ii) Action elements to which contact points belong

(iii) Time-action forms of contact points

(iv) Connecting form in which path elements connect contact points

Factors i, ii and iii must be determined regarding each contact point, while factor iv may be determined by expressing the connecting form algebraically resorting to the method explained in Part I. Consequently, what must be considered anew are the forms of expressing factors i, ii and iii, and they may be taken as following: For letter M or B that designates the type of contact point, place the name of action element to which it belongs at the left lower side of the letter and τ which shows the starting time of action at the right lower side as subscripts.

The contact point is not, within the time required for consideration, limited to only one action, but is generally subjected to several operation and release. As a method of representing one contact point having such a time-action form, it is simpler to replace the contact point by a group of contact points each of which acts only once. This method may be explained as follows:

In case of M contact point—For the sake of simplicity four kinds of acting forms shown in the left side of Fig. 13 are taken. Each M contact point having such a acting form may be replaced respectively by a supposed simple partial path shown in the right side of Fig. 13, and then is expressed by (24) where the action element is A.

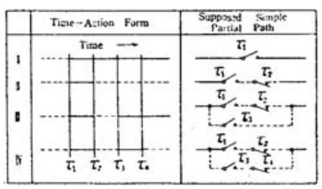

Fig. 13.

$$\left.\begin{array}{ll} \text{i.} & aM\tau_1 \\ \text{ii.} & aM\tau_1+aB\tau_2 \\ \text{iii.} & (aM\tau_1+aB\tau_2)aM\tau_3 \\ \text{iv.} & (aM\tau_1+aB\tau_2)(aM\tau_3+aB\tau_4) \end{array}\right\} \cdots(24)$$

In general, when a M contact point operates n times and releases n times, it may be expressed by (25).

$$(aM\tau_1+aB\tau_2)(aM\tau_3+aB\tau_4) \cdots\cdots(aM\tau_{2n-1}+aB\tau_{2n}) \cdots\cdots(25)$$

When a M contact point operates n times and releases $(n-1)$ times, it may be expressed by (26).

$$(aM\tau_1+aB\tau_2)(aM\tau_3+aB\tau_4) \cdots\cdots(aM\tau_{2n-1}) \cdots\cdots(26)$$

In case of B contact point—Considering in the same way as in the case of M contact point, B contact points shown in Fig. 14 can be expressed by (27).

$$\left.\begin{array}{ll} \text{i.} & aB\tau_1 \\ \text{ii.} & aB\tau_1\cdot aB\tau_2 \\ \text{iii.} & aB\tau_1\cdot aM\tau_2+aB\tau_3 \\ \text{iv.} & aB\tau_1\cdot aM\tau_2+aB\tau_3\cdot aM\tau_4 \end{array}\right\} \cdots(27)$$

When a B contact point operates n times and releases n times, it may be expressed by (28).

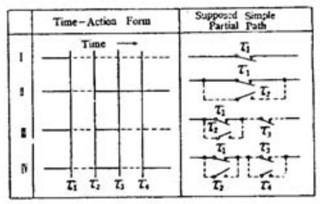

Fig. 14.

$$aB\tau_1\cdot aM\tau_2+aB\tau_3\cdot aM\tau_4+ \cdots\cdots+aB\tau_{2n-1}\cdot aM\tau_{2n} \cdots\cdots(28)$$

When a B contact point operates n times and releases $(n-1)$ times, it may be expressed by (29).

$$aB\tau_1\cdot aM\tau_2+aB\tau_3\cdot aM\tau_4+ \cdots\cdots+aB\tau_{2n-1} \cdots\cdots(29)$$

Thus it is shown that, for a simple partial path containing only one contact point, the type of contact point, the action element to which a contact point belongs and the time-action form, can all be expressed algebraically.

As a simple partial path having several contact points may be considered a composite simple partial path which has, as components, simple partial paths each containing only one contact point, it can be expressed algebraically by the method described in Part I.

II—2. LAW OF ELIMINATION REGARDING THE TYPES AND ACTING ORDER OF CONTACT POINTS

The acting impedance of simple partial path becomes either zero or infinite depending upon the action of contact points contained within it, but this does not mean that all actions of all contact points affect the acting function of the simple partial path as a whole. That is, depending upon the types, acting order and connection form of contact points, actions of some of contact points do not affect it at all. Such contact points may be eliminated from the simple partial path, and the general law regarding such elimination will be given.

To designate the acting order, two letters, α and β will be employed. With β to indicate a later acting order than α, the following equations are obtained:

i. $M_\alpha + M_\beta = M_\beta$ ························(30)

ii. $B_\alpha + B_\beta = B_\alpha$ ························(31)

iii. $M_\alpha \cdot M_\beta = M_\alpha$ ························(32)

iv. $B_\alpha \cdot B_\beta = B_\beta$ ························(33)

v. $\left.\begin{array}{l} B_\alpha + M_\beta = p \\ (B_\alpha + M_\beta)X = X \\ B_\alpha + M_\beta + X = p \end{array}\right\}$ ··················(34)

vi. $\left.\begin{array}{l} M_\alpha \cdot B_\beta = s \\ M_\alpha \cdot B_\beta \cdot X = s \\ M_\alpha B_\beta + X = X \end{array}\right\}$ ··················(35)

where X is an arbitrary simple partial path.

These equations show the case of two contact points. When there are many contact points these equations are to be applied in an orderly manner.

II—3. SOME EXAMPLES OF EQUIVALENT TRANSFORMATION

Let us now illustrate the equivalent transformation regarding the internal structure of some simple partial paths by applying the principles mentioned above.

Example 1. The simple partial path shown in Fig. 15.

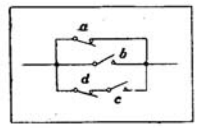

Fig. 15.

Assuming the acting order of four contact points to be the order of a, b, c and d, and these contact points to operate just once, the acting impedance Z of the simple partial path shown in Fig. 15 will be

$$Z = aB\tau_1 \cdot bM\tau_2(cM\tau_3 + dB\tau_4)$$
$$= aB\tau_1(bM\tau_2 \cdot cM\tau_3 + bM\tau_2 \cdot dB\tau_4)$$
$$= aB\tau_1 \cdot bM\tau_2$$

Thus, the contact points c and d can be entirely eliminated and the original simple partial path is shown to be equivalent to a simple partial path having a and b in parallel only.

Also, if the acting order is taken in the order of a, c, b and d, by similar operation,

$$Z = aB\tau_1 \cdot cM\tau_2$$

that is, b and d are entirely eliminated and the original simple partial path is equivalent to a simple partial path having a and c in parallel only.

Example 2. The simple partial path shown in Fig. 16.

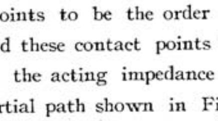
Fig. 16.

The simple partial path shown in Fig.

16 is first equivalently transformed into a simple partial path shown in Fig. 17 by the law of parallel separation and then the treatment is performed as before.

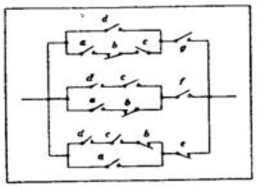

Fig. 17.

To begin with, when the acting order is taken in the order of a, b, c, d, e, f and g, and each contact points is considered to operate just once, the acting impedance will be

$$Z=aM\tau_1+eB\tau_5\cdot fM\tau$$

and the very complicated simple partial path shown in Fig. 16 becomes equivalent to a very simple one shown in Fig. 18.

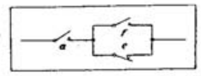

Fig. 18.

If the acting order is taken in the order of g, f, e, d, c, b and a and each contact point is considered to operate only once, the acting impedance will be

$$Z=dM\tau_4$$

and the simple partial path shown in Fig. 16 becomes equivalent to a simple one containing only one contact point d.

By the above illustration it will be easily understood how the acting function of a given simple partial path may be varied merely by changing the acting order of contact points. Also by such equivalent transformations it is very easy to see through the acting function of a given simple partial path and to obtain the simplest equivalent simple partial path.

PART II

This section gives the summaries of articles appearing in the Journal of the Institute of Electrical Communication Engineers of Japan published in Tokyo in the Japanese language. The following resumes are extracts from the April, May and June Journals of 1937.

THE THEORY OF FOUR-TERMINAL PASSIVE NETWORKS IN RELAY CIRCUIT

Akira Nakasima, Member
(Nippon Electric Co., Ltd., Tokyo)

This paper discusses the form of algebraic expression and properties of the transfer impedance of four-terminal passive networks in the relay circuit.

A four-terminal passive network is composed of contact points and simple path elements which connect the contact points, and transmits energy from one pair of terminals to the other. The energy transmission characteristic, which is controlled by actions of the contact points, is of all-or-nothing kind with lapse of time, and the transfer impedance Z_t has such a meaning as shown in Fig. 1.

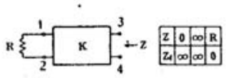

Fig. 1.

In order to express algebraically the transfer impedance in a general form and to know its properties, a general basic form of four-terminal network must be found. For this, an equivalent transformation from radial type to polygonal type complex partial path was first considered, and it was shown that Fig. 2 can be equivalently transformed to Fig. 3, provided that A_1, A_2 etc. are quite arbitrary simple partial paths.

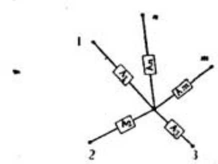

Fig. 2.

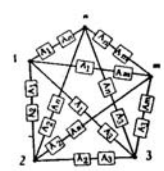

Fig. 3.

Moreover, such $\Delta-Y$ transformation as shown in Fig. 4 was demonstrated as being possible. These properties are useful for equivalent tranformation of bridge type simple partial paths or of more complicated types.

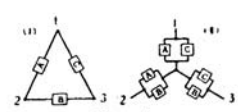

Fig. 4.

From the above-mentioned proofs it was shown that the four-terminal network, however complicated it may be, can be equivalently transformed into a complete quadrangle four-terminal network as shown in Fig. 5, and this was taken as the general basic form.

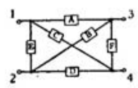

Fig. 5.

In considering this basic form, the general algebraic expression of the transfer impedance Z_t was obtained as follows:

$$Z_t = Z_s + \overline{Z_p} \quad \cdots\cdots(1)$$

in which

Z_s = impedance viewed from one pair of terminals provided the other pair is shortened,

Z_p = impedance viewed from the same pair of terminals as in the case of Z_s, provided the other pair is opened, and

$\overline{Z_p}$ = inverse impedance with regard to Z_p.

Applying relation (1) to the case shown in Fig. 5, relations (2) and (3) were obtained.

$$Z_t = AB + CD + \bar{A}\bar{C} + \bar{B}\bar{D} + \bar{E} + \bar{F} \quad \cdots(2)$$

$$Z_t = (A+D)(B+C) + (\bar{A}+\bar{D})(\bar{B}+\bar{C}) + \bar{E} + \bar{F} \quad \cdots\cdots(3)$$

As the expression shown in the right side of (3) has a symmetrical form with respect to Fig. 5, we know that the reciprocity theorem also holds for transmission characteristics of four-terminal passive networks in the relay circuit.

Considering the case of several arbitrary four-terminal passive networks being connected in cascade form, it was proven that the resultant transfer impedance is equal to the total sum of all transfer impedances, each of which belongs to each component network respectively and is not affected by the order of their connection.

Taking lattice type networks as special cases of Fig. 5, several interesting properties were found. These are shown in Fig. 6, Fig. 7, Fig. 8 and Fig. 9. The equal signs in the Figures mean that the transfer impedances of the networks are equal.

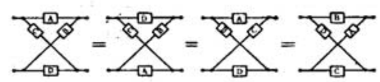

Fig. 6.

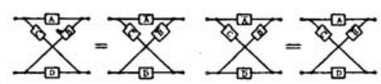

Fig. 7.

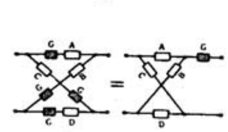

Fig. 8.

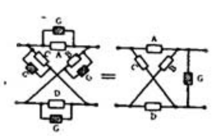

Fig. 9.

Finally, several forms of equivalent four-terminal passive networks with regard to transfer impedance were shown.

ALGEBRAIC EXPRESSIONS RELATIVE TO SIMPLE PARTIAL PATHS IN THE RELAY CIRCUIT

Akira Nakasima, Member

(Nippon Electric Co., Tokyo)

SYNOPSIS

This paper is intended to find a mathematical basis upon which problems regarding simple partial paths in relay circuit can be treated algebraically.

First, the acting impedance of a simple partial path is considered as a set of infinite-impedance points, and the one-to-one correspondence between any simple partial path and its corresponding set is explained.

Next, theorems and expressions used in the theory of set are applied to problems regarding simple partial paths to find their various interesting characters.

I. ACTING IMPEDANCE CONSIDERED BY THE NOTION OF SET

The acting impedance of simple partial path, in general, varies with time in such a way that its impedance value takes either one of two values, zero and infinity, at a time, as shown in Fig. 1.

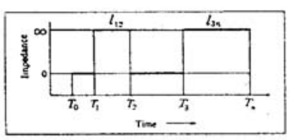

Fig. 1.

Fig. 1 shows that a time-impedance form of the acting impedance can be uniquely determined by knowing only either one of two forms, one regarding zero-impedance and the other regarding infinite-impedance. Hence, only the time-impedance form regarding infinite value will be considered in the following discussion.

In the example shown in Fig. 1, the acting impedance presents infinite value in two time regions (T_1T_2) and (T_3T_n). Line elements 1_{12} and 1_{3n} which represent this time-impedance form can be considered as a set of infinite-impedance points each of which holds a one-to-one correspondence with each time point contained in the time regions (T_1T_2) and (T_3T_n). This set of points is denoted by a set $\mathfrak{A}$.

Now we take a set of all infinite-impedance points each of which corresponds to each time point contained in the complete time region (T_0T_n), and this is denoted by "the complete set" $\mathfrak{P}$. It is clear that the above-mentioned set $\mathfrak{A}$ is "a subset" regarding the complete set $\mathfrak{P}$.

Next let us consider a set which has none of the infinite-impedance points contained in the complete set $\mathfrak{P}$ and this is denoted by "the empty set" $\mathfrak{S}$. A set which is derived by subtracting set $\mathfrak{A}$ from the complete set $\mathfrak{P}$ is also taken and this is denoted by "the complimentary set" $\overline{\mathfrak{A}}$.

From the above considerations we know that the following one-to-one correspondence exists between simple partial paths and sets of infinite-impedance points, as far as the acting impedance is concerned.

This condensed translation is about half the length of the original paper which appears in Japanese in the Journal of the Institute of Electrical Communicarion Engineers of Japan. No. 173, August, 1937.

A simple partial path "p" which is always open in the complete time region
...............Complete set $\mathfrak{P}$

A simple partial path "s" which is always closed in the complete time region
..................Empty set $\mathfrak{S}$

Arbitrary simple partial path "A"
..............................Set $\mathfrak{A}$

A simple partial path in double-inverse relation with A
.........Complimentary set $\overline{\mathfrak{A}}$

Any arbitrary simple partial path may be considered as a set which corresponds to its time-impedance form. Theorems and expressions developed in the theory of set may, therefore, be applied to acting impedance problems of simple partial paths. This is tbe reason why we can use the theory of set as the mathematical basis to treat algebraically the acting impedance problems of simple partial paths.

In the following discussion, simple partial paths are denoted by English letters and sets representing them are denoted by corresponding German letter.

II. COMPARISON OF ACTING IMPEDANCES OF TWO ARBITRARY SIMPLE PARTIAL PATHS

(1) Relations Expressed by Equal Sign =

When two simple partial paths, A and B, have the same acting impedance, the following expression is used: $A=B$. This means that their corresponding sets $\mathfrak{A}$ and $\mathfrak{B}$ are equal, that is, $\mathfrak{A}=\mathfrak{B}$. If $\mathfrak{A}=\mathfrak{B}$, then $\overline{\mathfrak{A}}=\overline{\mathfrak{B}}$ from the theory of sets. Therefore, as relations between two arbitrary simple partial paths A and B, two expressions $A=B$ and $\overline{A}=\overline{B}$ have the same meaning.

We know thus that there exists the following property with regard to acting impedance in the case of simple partial paths related by the equal sign:

When two arbitrary simple partial paths are equal, the two simple parthial paths in double-inverse relation with them respectively are also equal. ......(1)

Considering this from the standpoint of equational operation, it may be declared that,

In an equal expression, the equality holds also true if both the left and right hand sides are respectively doubly inversed. ......(2)

(2) Relations Expressed by Unequal Signs $\supset$ and $\subset$

In the theory of set, set $\mathfrak{B}$ is called a subset of another set $\mathfrak{A}$ when all elements of $\mathfrak{B}$ are contained in $\mathfrak{A}$, and the relation between $\mathfrak{A}$ and $\mathfrak{B}$ is expressed by signs $\supset$ and $\subset$, as follows: $\mathfrak{A}\supset\mathfrak{B}$ or $\mathfrak{B}\subset\mathfrak{A}$. When $\mathfrak{A}$ contains more different elements than those contained in $\mathfrak{B}$, $\mathfrak{B}$ is called a true subset of $\mathfrak{A}$. Only the true subset is treated in this paper.

From relations developed in the theory of set, we have the following relations regarding any arbitrary simple partial paths A and B:

$p\supset A$ ......(3)

$s\subset A$ ......(4)

If $A\supset B$, then $\overline{A}\subset\overline{B}$ ......(5)

If $A\supset\overline{B}$, then $\overline{A}\subset B$ ......(6)

The physical meaning of relation (5) will be clearly understood from one example shown in Fig. 2 where solid lines and dotted lines show zero-impedance and infinite-impedance respectively.

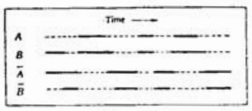

Fig. 2,

Regarding relations between two arbitrary simple partial paths, we know that there exists the following property:

When two arbitrary simple partial paths are related with each other by unequal sign, the relation between the two simple partial paths respectively in double-inverse relation with them is also expressed by the unequal sign the direction of which is reversed. ....(7)

Considering (7) from the standpoint of equational operation, it may be declared that,

In an unequal equation, the direction of unequal sign is reversed if both the left and right hand sides are respectively doubly inversed. ....(8)

III. EXPRESSIONAL FORMS AND RELATIONS OF COMPOSITE SIMPLE PARTIAL PATHS

(1) Meaning of Sum and Product of Sets

When set $\mathfrak{B}$ is composed in such a manner that it contains all elements contained in several sets, $\mathfrak{M}_1$, $\mathfrak{M}_2$,......and $\mathfrak{M}_n$, set $\mathfrak{B}$ is expressed by a form of sum as follows:

$$\mathfrak{B}=\mathfrak{M}_1+\mathfrak{M}_2+\cdots\cdots+\mathfrak{M}_n \quad \cdots\cdots\cdots\cdots\cdots(9)$$

When set $\mathfrak{D}$ is composed in such a manner that it contains only common elements among elements contained respectively in the several sets, $\mathfrak{M}_1$, $\mathfrak{M}_2$,......and $\mathfrak{M}_n$, set $\mathfrak{D}$ is expressed by a form of product as follows:

$$\mathfrak{D}=\mathfrak{M}_1\mathfrak{M}_2\cdots\cdots\mathfrak{M}_n \quad \cdots\cdots\cdots\cdots\cdots(10)$$

Now let us consider the meaning of expressions (9) and (10) with reference to the case of composite simple partial paths. As mentioned in section I, we treat the time-impedance form of acting impedance as a set of infinite-impedance points. Hence it will be easily understood that a composite simple partial path composed by series and parallel connections of several simple partial paths can be expressed by relation (9) and by relation (10) respectively.

In addition we know that the following distributive law holds also true in the theory of set:

$$\left.\begin{aligned}&\mathfrak{M}(\mathfrak{M}_1+\mathfrak{M}_2+\cdots\cdots\mathfrak{M}_n)\\&=\mathfrak{M}\mathfrak{M}_1+\mathfrak{M}\mathfrak{M}_2+\cdots\cdots+\mathfrak{M}\mathfrak{M}_n\end{aligned}\right\}\cdots(11)$$

Thus, we know the expressional form of composite simple partial paths made by series and parallel connections of several simple partial paths, and that the associative, commutative and distributive laws all hold true for its equational operation.

(2) Relations with Regard to Complimentary Set

For any arbitrary set and its complimentary set the following relations exist:

$$\left.\begin{aligned}&\overline{(\overline{\mathfrak{A}})}=\mathfrak{A}\\&\overline{(\mathfrak{A}_1+\mathfrak{A}_2+\cdots\cdots+\mathfrak{A}_n)}=\overline{\mathfrak{A}}_1\overline{\mathfrak{A}}_2\cdots\cdots\overline{\mathfrak{A}}_n\\&\overline{(\mathfrak{A}_1\mathfrak{A}_2\cdots\cdots\mathfrak{A}_n)}=\overline{\mathfrak{A}}_1+\overline{\mathfrak{A}}_2+\cdots\cdots+\overline{\mathfrak{A}}_n\\&\mathfrak{A}+\overline{\mathfrak{A}}=\mathfrak{P}\\&\mathfrak{A}\overline{\mathfrak{A}}=\mathfrak{S}\end{aligned}\right\} \cdots\cdots\cdots\cdots(12)$$

These relations can be soon transformed

into relations regarding simple partial paths which are in double-inverse relation with each other, and they serve the equational treatment very much.

(3) Relations with Regard to Subset

For several sets which are mutually related as subsets, there exist the following relations:

$$\left.\begin{array}{ll}\text{If} & \mathfrak{M}_1\subset\mathfrak{M}_2\subset\cdots\cdots\subset\mathfrak{M}_n,\\ \text{then} & \mathfrak{B}=\mathfrak{M}_1+\mathfrak{M}_2+\cdots\cdots+\mathfrak{M}_n=\mathfrak{M}_n\\ \text{and} & \mathfrak{D}=\mathfrak{M}_1\mathfrak{M}_2\cdots\cdots\mathfrak{M}_n=\mathfrak{M}_1\end{array}\right\}\cdots\cdots(13)$$

$$\left.\begin{array}{l}\mathfrak{A}\mathfrak{B}+\mathfrak{A}=\mathfrak{A}\\ \mathfrak{A}(\mathfrak{A}+\mathfrak{B})=\mathfrak{A}\end{array}\right\}\cdots\cdots(14)$$

$$\left.\begin{array}{l}\mathfrak{A}+\mathfrak{A}+\cdots\cdots+\mathfrak{A}=\mathfrak{A}\\ \mathfrak{A}\mathfrak{A}\cdots\cdots\mathfrak{A}=\mathfrak{A}\end{array}\right\}\cdots\cdots(15)$$

If $\mathfrak{A}\supset\mathfrak{B}$ and $\mathfrak{B}\supset\mathfrak{C}$, $\mathfrak{A}\supset\mathfrak{B}\supset\mathfrak{C}$ $\cdots(16)$

If $\mathfrak{A}\supset\mathfrak{B}$ and $\mathfrak{A}\supset\mathfrak{C}$, $\mathfrak{A}+\mathfrak{C}\supseteqq\mathfrak{B}+\mathfrak{C}$ $\cdots\cdots(17)$

If $\mathfrak{A}\supset\mathfrak{B}$ and $\mathfrak{B}\subset\mathfrak{C}$, $\mathfrak{A}\mathfrak{C}\supseteqq\mathfrak{B}\mathfrak{C}$ $\cdots\cdots(18)$

The above relations can be used as the principle of elimination in case of equivalent transformation of simple partial paths.

IV. PRINCIPLE OF DUAL CORRESPONDENCE IN RELATIONS

Among the above-mentioned relations we find a very interesting property, that is, the existence of the principle of dual correspondence which enables us to derive another group of relations directly from a group of relations.

First, let us consider the relation (11). When the relation (11) is transformed by relations (2), (5) and (12), we have

$$\overline{\mathfrak{M}}+\overline{\mathfrak{M}}_1\overline{\mathfrak{M}}_2\cdots\cdots\overline{\mathfrak{M}}_n=(\overline{\mathfrak{M}}+\mathfrak{M}_1)(\overline{\mathfrak{M}}+\overline{\mathfrak{M}}_2)\cdots\cdots(\overline{\mathfrak{M}}+\overline{\mathfrak{M}}_n)$$

Eliminating the head bar from this eqation, we have

$$\mathfrak{M}+\mathfrak{M}_1\mathfrak{M}_2\cdots\cdots\mathfrak{M}_n=(\mathfrak{M}+\mathfrak{M}_1)(\mathfrak{M}+\mathfrak{M}_2)\cdots\cdots(\mathfrak{M}+\mathfrak{M}_n)\cdots\cdots(19)$$

The equation (19) corresponds dually with the equation (11), and shows a new distributive law which does not hold true in primary algebra. If the significance of equation (19) is considered with respect to three arbitrary simple partial paths A, B and C, the following is obtained:

$$AB+C=(A+C)(B+C)$$

This is the very relation used in series to parallel equivalent transformation of composite simple partial paths.

Thus, applying double-inverse transformation to any given relation, we have a new relation which corresponds dually with the original one, and many examples can be found in the above-mentioned relations.

From these facts, we may say that,

When an equal relation holds true between two arbitrary composite simple partial paths, then another equal relation holds true, which relation is derived from the original expression by changing the signs of the sum and product to the signs of product and sum respectively. ……(20)

When an unequal relation holds true between two arbitrary composite simple partial paths, then another unequal relation holds, which relation is derived from the original expression by changing the signs of the sum and product to the signs of product and sum respectively, and by reversing the direction of the unequal sign. ……(21)

V. SOLUTIONS OF ACTING IMPEDANCE EQUATIONS OF SIMPLE PARTIAL PATHS

In the former sections, it has been shown that any arbitrary composite simple partial path composed by series and parallel connections of several component simple partial paths can be expressed algebraically in forms of sum and product of the components; and what manner of equational operation is available in such algebraic expressions, has been clearly shown.

Now let us consider how to find an unknown component simple partial path when the composite one is known. This problem is very difficult, but some attempt may be made here. In the following equations the unknown component simple partial path is denoted by X.

(i) If $X+A=A$, then $X \subseteqq A$ ······(22)

(ii) If $XA=A$, then $X \supseteqq A$ ·········(23)

(iii) If $X+A=B$,

then $\overline{A}B \subset X \subset A+B$ ······(24)

(iv) If $XA=B$,

then $\overline{A}+B \supset X \supset AB$ ······(25)

These solutions determine only some parts of the acting impedance of X. The other parts remain indefinite. For instance, if the acting impedance of A, B and X are as shown in Fig. 3, then the solution given by (24) determines the acting impedance of X only in the time regions (T_2T_3), (T_3T_4), (T_6T_7), (T_8T_9) and (T_9T_{10}) as shown by $\overline{A}B$ X and $A+B$ X in the same Figure.

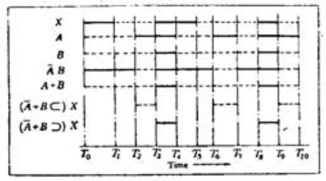

Fig. 3.

(v) If $AX+B=C$
then $\overline{B}C \subset AX \subset B+C$,
and $(A+B)C \subset X+B \subset \overline{A}\overline{B}+C$ ······(26)

(vi) If $(A+X)B=C$,
then $AB+C \supset BX \supset (\overline{A}+\overline{B})C$
and $\overline{B}+C \supset A+X \supset BC$ ······(27)

(vii) If $X+A=B$ and $X+\overline{A}=C$
then $X=BC$ ······(28)

(viii) If $XA=B$ and $X\overline{A}=C$
then $X=B+C$ ······(29)

(ix) If $X+A=B$ and $XA=C$
then $X=B(C+\overline{A})$ ······(30)

PART I

THE THEORY OF TWO-POINT IMPEDANCE OF PASSIVE NETWORKS IN THE RELAY CIRCUIT

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Synopsis

This paper is intended to develop a general theory regarding the two-point impedance of passive networks in relay circuits by the theory of set.

First, a law of superposition is found with regard to the two-point impedance, and according to this law it is analytically proven that the law of parallel separation holds always true with regard to the equivalent transformation of composite simple partial paths.

Next, a method of obtaining a two-point impedance of a composite network composed of several networks, whose impedance characteristics are known, is given and a principle of network separation regarding the two-point impedance is found.

Finally, the mutual relations and properties of various two-point impedances of a four-terminal passive network are shown.

Contents

This is a condensed translation of the original paper which appears in Japanese in the Journal of the Institute of Electrical Communication Engineers of Japan, No. 177, December 1937 and 178, January, 1938.

I. Method of obtaining the Acting Impedance between any Two Points in a Network—"Law of Superposition"

Let us consider how the acting impedance between any two points in a composite network may be expressed as a function of various acting impedances of component simple partial paths in a composite network.

This problem is first considered with regard to a composite network shown, as an example, in Fig. 1. In this network there are ten simple partial paths, $A, B, C, D, E, F, G, H, J$ and K. Let the acting impedance of these simple partial paths be known and be denoted by letters $A, B, C, D, E, F, G, H, J$ and K respectively.

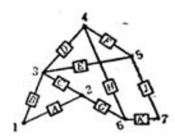

Fig. 1.

Now, the acting impedance between point 1 and point 7 in this composite network is taken into consideration. In order to obtain this acting impedance, let us first find out the number of paths from point 1 to point 7, such paths passing through any one connecting point only once. We find that there are thirteen such paths. Since the acting impedance of a path can be expressed as a sum of the various acting impedances of all simple partial paths in such a path, the acting impedances, Z_1 to Z_{13}, of these thirteen paths are expressed as follows:

$$\left.\begin{aligned} Z_1&=A+G+K\\ Z_2&=A+G+H+F+J\\ Z_3&=A+G+H+D+E+J\\ Z_4&=A+C+D+H+K\\ Z_5&=A+C+D+F+J\\ Z_6&=A+C+E+J\\ Z_7&=A+C+E+F+H+K\\ Z_8&=B+C+G+K\\ Z_9&=B+C+G+H+F+J\\ Z_{10}&=B+E+J\\ Z_{11}&=B+E+F+H+K\\ Z_{12}&=B+D+F+J\\ Z_{13}&=B+D+H+K \end{aligned}\right\}\cdots\cdots\cdots(1)$$

According to the notion of set, it is clear that the resultant acting impedance Z between point 1 and point 7 can be expressed by the product of the acting impedances of these thirteen paths, that is:

$$\begin{aligned} Z=&(A+G+K)\,(A+G+H+F+J)\\ &\times(A+G+H+D+E+J)\,(A+C+D+H+K)\\ &\times(A+C+D+F+J)\,(A+C+E+J)\\ &\times(A+C+E+F+H+K)\,(B+C+G+K)\\ &\times(B+C+G+H+F+J)\,(B+E+J)\\ &\times(B+E+F+H+K)\,(B+D+F+J)\\ &\times(B+D+H+K)\ \cdots\cdots\cdots\cdots\cdots\cdots(2) \end{aligned}$$

The foregoing considerations and the expressional form above give a fundamental principle, namely, "*The Law of Superposition regarding Two-Point Impedances.*" This law can be expressed generally as follows:

The acting impedance between any two points in any passive network is equal to the total product of all the acting impedances of the various paths between such two points—"various paths" here referred to being only those paths in each of which no point is passed more than once.

This law of superposition can be algebraically expressed as follows:

$$Z=Z_1\,Z_2\cdots\cdots\cdots Z_n\ \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

where,

Z = acting impedance between any two points

n = number of paths (between such two points) in each of which no point is passed more than once, and

Z_n = acting impedance of the nth path.

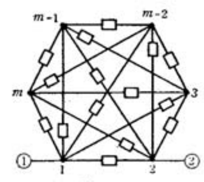

Fig. 2.

The total number of paths (such as those defined above) between any two points in the complete m-angle network shown in Fig. 2 is indicated by the following expression:

$$N_m=1+(m-2)\times N_{m-1}\ \cdots\cdots\cdots\cdots\cdots(4)$$

where,

N_m = total number of paths between any two points in a complete m-angle network.

From expression (4) we obtain the next expression:

$$\begin{aligned} N_m=&1+(m-2)+(m-2)(m-3)\\ &+(m-2)(m-3)(m-4)\\ &+(m-2)(m-3)(m-4)(m-5)\\ &+(m-2)(m-3)(m-4)(m-5)\\ &(m-6)+\ \cdots\cdots\cdots\cdots\cdots\cdots(5) \end{aligned}$$

II. Law of Parallel-Separation and its Proof

Let us now attempt to prove that the law of parallel-separation generally holds true regarding the equivalency of an acting impedance between any two points in an arbitrary passive network. As an arbitrary passive network, it is sufficient to take any complete polyangular network. In the following, therefore, the acting impedance between point 1 and point 2 in the network shown in Fig. 2 is treated.

The necessary and sufficient number of paths from point 1 to point 2 passing through a branch (1, $m-1$) is N_{m-1}, which is given by the expression (5). The resultant acting impedance $Z_{1,\,m-1}$ presented by these paths is shown as follows according to the law of superposition:

$$Z_{1,\,m-1}=({}_1\mathfrak{z}_{m-1}+\mathfrak{z}_1)\,({}_1\mathfrak{z}_{m-1}+\mathfrak{z}_2)\cdots\cdots({}_1\mathfrak{z}_{m-1}+\mathfrak{z}_n)$$

where,

${}_1\mathfrak{z}_{m-1}$ = acting impedance of the branch $(1,\ m-1)$,

$\mathfrak{z}_1,\ \mathfrak{z}_2,\cdots\cdots\mathfrak{z}_n$ = respective acting impedance of paths from point $(m-1)$ to point 2 in a network which is derived from the original network by eliminating all branches gathering at point 1

and

$n=N_{m-1}$

By transforming the right hand side of this expression by the distributive law, the following is obtained:

$$Z_{1,\,m-1}={}_1\mathfrak{z}_{m-1}+\mathfrak{z}_1\times\mathfrak{z}_2\cdots\cdots\times\mathfrak{z}_n \quad\cdots\cdots(6)$$

When an acting impedance between point $(m-1)$ and point 2 in a network, which is derived from the original network shown in Fig. 2 by eliminating all the branches gathering at point 1, is denoted by $Y_{m-1,\,2}$, the following relation holds true:

$$Y_{m-1,2}=\mathfrak{z}_1\times\mathfrak{z}_2\times\cdots\cdots\mathfrak{z}_n \quad\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

From expression (6) and relation (7), the acting impedance $Z_{1,\,m-1}$ is expressed as follows:

$$Z_{1,\,m-1}={}_1\mathfrak{z}_{m-1}+Y_{m-1,\,2} \quad\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

The acting impedance Z between point 1 and point 2 in the original network can be expressed as a form of total product of $Z_{1,\,2}$, $Z_{1,\,3}$, $\cdots\cdots Z_{1,\,m-1}$ and $Z_{1,m}$, according to the law of superposition mentioned above, that is,

$$Z=({}_1\mathfrak{z}_2+Y_{2,\,2})\,({}_1\mathfrak{z}_3+Y_{3,2})\cdots\cdots\cdots\cdots({}_1\mathfrak{z}_{m-1}+Y_{m-1,\,2})\,({}_1\mathfrak{z}_m+Y_{m,\,2}) \quad\cdots\cdots\cdots\cdots(9)$$

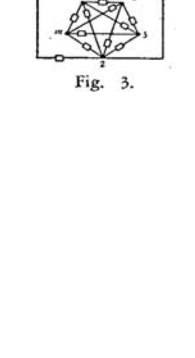

Fig. 3.

This expression (9) shows that the acting impedance between point 1 and point 2 in the original network shown in Fig. 2 is equal to the acting impedance between point 1 and point 2 in the derived network shown in Fig. 3, this latter network having a graphical form composed by separating the original network into a parallel form with respect to the branches gathering at point 1. This fact proves that the law of parallel separation generally holds true with regard to the two-point impedance in an arbitrary passive network.

By the application of this law, it becomes very easy to express algebraically the two-point impedance in a passive network, however complicated its form may be.

III. Acting Impedance between Two Points of a Composite Network composed of Two Networks connected at Two Points in Each Network

Let us obtain the acting impedance viewed from any two points of either one of the two networks which are connected at two points of each to compose a composite network.

In Fig. 4, K_1 and K_2 are any passive networks whose impedance characteristics are known. They are connected at points c, d and e, f. Let us consider the impedance Z viewed from any two points, a and b, in K_1.

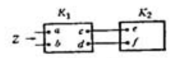

Fig. 4.

Let:

Z_1 = Impedance between a and b of K_1

Z_{1s} = Impedance between a and b of K_1 in the case of c and d being short-circuited.

Z_2 = Impedance between e and f of K_2

Z = Impedance between a and b in the composite network,

and we get the following:

$$Z=Z_{1s}+Z_1\cdot Z_2 \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

This is illustrated in Fig. 5. The arrow

Fig. 5.

sign indicates the acting impedance between two points to be considered.

1. Resultant Transfer Impedance in the Case of Two 4-Terminal Networks connected in Cascade

Let us take the case of any two 4-terminal networks, K_1 and K_2, connected in cascade form as in Fig. 6.

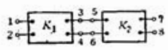

Fig. 6.

Let:

Z_{1p}=Impedance viewed from terminals 1 and 2 of network K_1, with terminals 3 and 4 opened,

Z_{1s}=Impedance viewed from terminals 1 and 2 of network K_1, with terminals 3 and 4 short-circuited,

Z_{2p}=Impedance viewed from terminals 5 and 6 of network K_2 with terminals 7 and 8 opened,

Z_{2s}=Impedance viewed from terminals 5 and 6 of network K_2 with terminals 7 and 8 short-circuited,

Z_p = Impedance viewed from terminals 1 and 2 of composite network with terminals 7 and 8 opened,

Z_s = Impedance viewed from terminals 1 and 2 of composite network with terminals 7 and 8 short-circuited;

Z_{1t} = Transfer impedance of network K_1,

Z_{2t} = Transfer impedance of network K_2, and

Z_t = Transfer impedance of composite network.

Then, we get the following relations from equation (10):

$$Z_p = Z_{1s} + Z_{1p} \cdot Z_{2p}$$
$$Z_s = Z_{1s} + Z_{1p} \cdot Z_{2s}$$

Since

$$Z_t = Z_s + \overline{Z}_p,$$

we get

$$\begin{aligned} Z_t &= Z_s + \overline{Z}_p \\ &= (Z_{1s} + Z_{1p} \cdot Z_{2s}) + \overline{Z}_{1s}(\overline{Z}_{1p} + \overline{Z}_{2p}) \\ &= (Z_{1s} + \overline{Z}_{1p}) + (Z_{2s} + \overline{Z}_{2p}) \\ &= Z_{1t} + Z_{2t} \end{aligned}$$

That is, the transfer impedance of the composite network is equal to the sum of the transfer impedances of its component networks.

2. Composite Network composed by Iterative Connection of Networks having Equal Two-Point Impedance

Let us first consider the case of a composite network comprising two networks as given on the left side of Fig. 7.

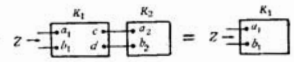

Fig. 7.

Making Z_1 and Z_2 as the impedances viewed respectively from the pair of terminals (a_1 and b_1) of network K_1 and from the pair of terminals (a_2 and b_2) of network K_2, making Z_{1s} as the impedance viewed from a_1 and b_1 of network K_1 with the pair of terminals c and d short-circuited, and making Z the impedance viewed from a_1 and b_1 in the composite network, we get from the original premise above,

$$Z = Z_{1s} + Z_1 \cdot Z_1 = Z_{1s} + Z_1$$

by substitution in equation (10) because $Z_1 = Z_2$. But, since,

$$Z_1 = Z_{1s} + Z_1$$

we get, by substituting this in the preceding equation,

$$Z = Z_1 \quad \cdots\cdots(11)$$

This equation is illustrated graphically in Fig. 7.

Although the result of the above resembles that of the case of connection by the iterative impedance basis in the transmission circuit theory, the above network system possesses considerable freedom with regard to the position of the connecting points.

We can summarize the foregoing by stating:

In a composite network comprising networks which are iteratively connected and whose two-point impedances are equal, the impedance of the composite network viewed from the two points of the first preceding network is equal to the two-point impedance of the respective component networks, regardless of the number of networks comprised in the composite network and regardless of the position of the two points in the preceding network to be connected by the following network.

A graphically illustrated example of this is shown in Fig. 8.

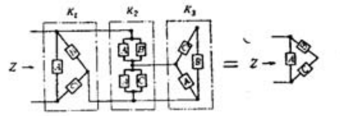

Fig. 8.

3. Network terminated by its Short-Circuited Impedance

In this case we have the relation shown in Fig. 9.

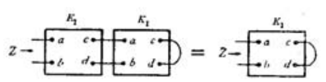

Fig. 9.

4. Network connected by a Simple Partial Path Equivalent to $\bar{Z}_1$ or $\bar{Z}_{1s}$

In this case we get the relations shown in Fig. 10 and Fig. 11.

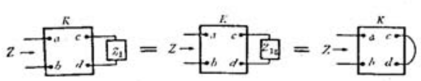

Fig. 10.

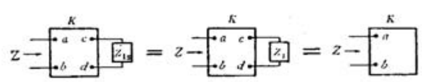

Fig. 11.

5. Network in which One Branch is Opened

Let us now consider the impedance between the two points a and b of network K in Fig. 12, in the case of the branch (whose impedance is ${}_c\mathfrak{z}_d$) between any two points (c and d) in the network being opened. To open the branch between c and d means the same as inserting in series to it a simple partial path having impedance ${}_c\bar{\mathfrak{z}}_d$.

Thus, we get the top graphical trans-

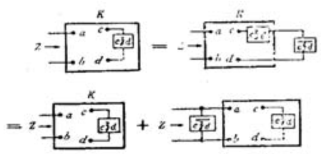

Fig. 12.

formation in Fig. 12, and if we transform the two-point connection with regard to ${}_c\bar{\mathfrak{z}}_d$, we get the lower graphical transformation.

From Fig. 12, the graphic equation in Fig. 13 can be obtained. K is an arbitrary network; a, b, c, and d are any points; and X is any simple partial path. Hence, we

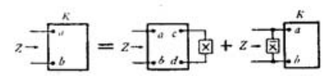

Fig. 13.

can hold:

The impedance, viewed from two points (a and b) of a network, is equal to the sum of the impedance of this network in the case of its any two points being connected with any simple partial path, and of the impedance of the network in the case of its two points a and b being connected with a simple partial path which is in double-inverse relation to the above simple partial path.

Furthermore, if both sides of the graphic equation in Fig. 13 are multiplied by X, we get the equation in Fig. 14. Considering this as a transformation in reverse, from the right to the left side, we can assert the following:

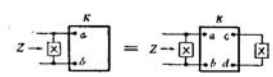

Fig. 14.

The impedance between two given points of a network is unaffected by the elimination of the simple partial path, which is contained in this network and is equivalent to that simple partial path between those two points.

This offers one method of elimination.

IV. Acting Impedance viewed from Two Points of a Network (in a Composite Network) to which Other Component Networks are connected at Two Points Respectively

Let us consider the problem of obtaining the impedance between any two points in a given network of a composite network in which the said given network is connected with any two points in each of the several other component networks. These networks may be of any networks but we assume that their impedance characteristics are as already known.

1. The Case of a Given Network being connected at Two Points of Each of Two Other Networks

When any two pairs of points in any given network are respectively connected to any other two networks X and Y, as indicated in the first preceding network in Fig. 15, the impedance Z viewed from any two points a and b in the said given network may be graphically expressed as by Fig. 15.

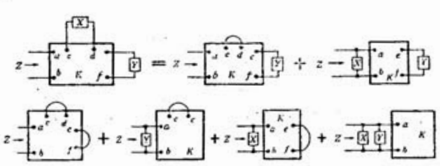

Fig. 15.

If in Fig. 15 $X=Y$, we get the relation shown in Fig. 16.

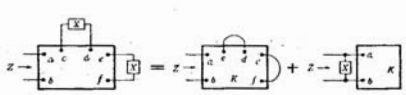

Fig. 16.

2. Transfer Impedance in the Case of Equivalent Simple Partial Paths being connected in Parallel with All Branches of a 4-Terminal Network

Let us prove that the transfer impedance of any network is equal to the sum of the transfer impedance of the network, in which the parallel simple partial paths included commonly in each branch of the original given network are eliminated, and the transfer impedance of the 4-terminal network which comprises the above parallel simple partial paths only as parallel elements.

Now, with X being the impedance of parallel simple partial path, we can express the impedances of a network (in whose original form parallel simple partial paths are included but are here eliminated) as follows, with its impedance symbols indicated by $'$ while those of the original network are indicated without it:

$$Z_s = Z_s'\cdot X,\quad Z_p = Z_p'\cdot X$$

$$Z_t = Z_s + \overline{Z}_p$$

$$= Z_s'\cdot X + \overline{Z}_p' + \overline{X}$$

$$= Z_s' + \overline{Z}_p' + \overline{X}$$

$$\therefore\quad Z_t = Z_t' + \overline{X}$$

3. Composite Network comprising Several Networks having Equal Two-Point Impedances

Let us consider here a simple case like that on the left side of the graphic equation in Fig. 17.

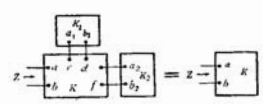

Fig. 17.

With the impedances, viewed from points (a_1 and b_1) and points (a_2 and b_2) of networks K_1 and K_2 respectively, being equal to the impedance viewed from the two points a and b in network K, we obtain the relation shown in Fig. 17. We can therefore hold:

The acting impedance, viewed from two given points of a given network, is equal to that viewed from the same two points of the composite network. in which the said given network is connected at any two points respectively with other component networks possessing two-points acting impedances equal to that of the said original given network.

V. Acting Impedance between Two Points of a Composite Network composed of Two Networks connected at Three or More Points in Each Network —— The Law of Network Separation

1. Composite Network comprising Two Networks connected at Three Points in Each

Let us consider the case of Fig. 18 (i). K_2 may be any network, but for the sake of convenience let us transform it into a complete triangular network, as shown in Fig. 18 (ii), which is equivalent to network

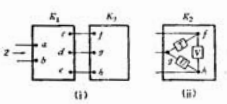

Fig. 18.

K_2 with regard to points f, g and h. By applying the results obtained in the preceding sections to the composite network of (i) in which K_2 is a complete triangular network, we get the transformation shown in Fig. 19.

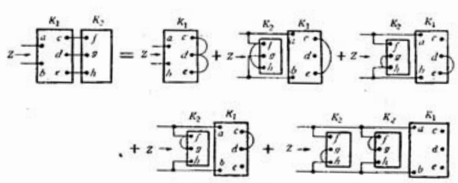

Fig. 19.

2. Composite Network comprising Two Networks connected at n Number of Points in Each

Let us take the case of the composite network in which two networks K_1 and K_2 are connected at n number of points in each, and in which the impedance viewed from two points of K_1 is to be obtained.

In this case, following the method explained in the preceding sub-section 1, we can get this impedance by first transposing network K_2 with a complete n-angular network (equivalent to K_2) having n number of connecting points as vertices, then applying to this the transformation method of the case of two-point connection form, and finally by reducing, to the original network K_2, the partial networks composed of the branches in the complete n-angular network.

Thus, with regard to the two-point impedance of composite networks generally, we have seen that with the laws given in sections II and III above it is possible to express it by separating the composite network into its component networks. Let this process be named the "*Law of Network Separation Regarding Two-Point Impedance.*"

VI. Mutual Relations of Various Two-Point Acting Impedances in a 4-Terminal Network

Taking only four points in any passive network, let us next consider the relations between their various two-point acting impedances. The reasons for limiting the number of points to four are that in this case the mutual relations are easy to obtain, and that their characteristics are directly useful when considering, for example, the problem of transfer impedance.

1. Extention of Two-Point Impedance

In the case under consideration, we generally have the following expression:

$$Z_{jk}=Z_{jk}(\overline{k,l,m})+Z_{jk}(\overline{j,l,m})+Z_{jk}(\overline{j,l},\ \overline{k,m})+Z_{jk}(\overline{j,m},\ \overline{k,l}) \qquad \cdots\cdots(12)$$

Fig. 20 illustrates this graphically.

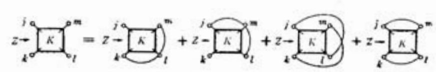

Fig. 20.

2. Mutual Relations of Various Impedances viewed from the Same Two Points

Let us examine the mutual relations between the impedances viewed from the same two points (j and k for example), in case of the four points being variously short-circuited.

With regard to two-point impedances generally, the impedance in the case of any two points in the network being short-circuited is in the relation of subset to that of the case of no short-circuit. For example,

$Z_{jk} \supset Z_{jk}(\overline{l, m}) \supset Z_{jk}(\overline{l, m, k})$

Next, if we consider the mutual relations of various two-point impedances of networks possessing common points of short-circuit, we get the relations shown in Fig. 21. If we consider this together with the expression given above showing the relation of the subset, we get the following general property:

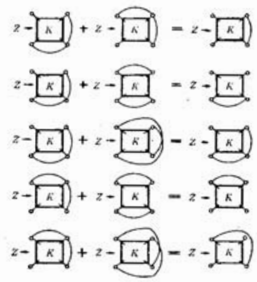

Fig. 21.

When considering the acting impedance between two of any four points in any passive network, the sum total of the two-point impedances in the case of these four points being variausly short-circuited is equal to the two-point impedance of the case of the common short-circuit points only being short-circuited, if such common points of short-circuit do exist.

Furthermore, the relations shown in Fig. 22 and Fig. 23 generally exist.

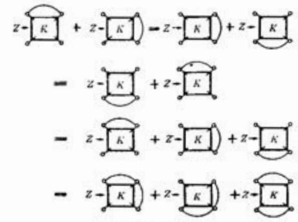

Fig. 22.

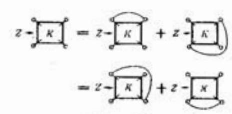

Fig. 23.

3. Mutual Relations of Impedances between Various Two Points

The following properties may be given:

$$\left.\begin{aligned} & Z_{jk}+Z_{jl}+Z_{jm}=Z_{jk}+Z_{jl}+Z_{km} \\ &=Z_{jk}+Z_{jl}+Z_{lm}=Z_{jl}+Z_{jm}+Z_{km} \\ &=Z_{jl}+Z_{jn}+Z_{kl}=Z_{jm}+Z_{jk}+Z_{kl} \\ &=Z_{jm}+Z_{jk}+Z_{lm}=Z_{jk}+Z_{km}+Z_{kl} \\ &=Z_{jk}+Z_{km}+Z_{lm}=Z_{km}+Z_{kl}+Z_{jl} \\ &=Z_{km}+Z_{kl}+Z_{jm}=Z_{kl}+Z_{jk}+Z_{lm} \\ &=Z_{kl}+Z_{jl}+Z_{lm}=Z_{jl}+Z_{lm}+Z_{km} \\ &=Z_{lm}+Z_{kl}+Z_{jm}=Z_{km}+Z_{jm}+Z_{lm} \end{aligned}\right\} \cdots\cdots(13)$$

$$Z_{jk}+Z_{jl} \supset Z_{kl} \quad \cdots\cdots(14)$$

$$\left.\begin{aligned} & Z_{jm}+Z_{kl}+\overline{Z_{jk}} \\ &=Z_{jm}+Z_{kl}+\overline{Z_{jl}} \\ &=Z_{jm}+Z_{kl}+\overline{Z_{km}} \\ &=Z_{jm}+Z_{kl}+\overline{Z_{lm}} \end{aligned}\right\} \cdots\cdots(15)$$

PART I

THE TRANSFER IMPEDANCE OF FOUR-TERMINAL PASSIVE NETWORKS IN THE RELAY CIRCUIT

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Synopsis

This paper is intended to develop a theory regarding the transfer impedance which indicates the transmitting function of the energy through four-terminal passive networks in the relay circuit, taking into consideration of its phase characteristics as well as its quantitative characteristics.

First, the meanings of the positive and negative transfer impedances are explained, and expressional forms these transfer impedances in terms of various two-point acting impedances of a four-terminal network are found out. Next, the relationship between these transfer impedance and the absolute transfer impedance, which is considered with regard to the quantitative characteristics only, is shown, thereby giving a theoretical proof to the equation, $Z_t = Z_s + \overline{Z}_p$, shown in my previous paper. And further, a method of obtaining the resultant transfer impedance in the case of several four-terminal networks being connected in cascade form is given. Finally, several interesting properties regarding the transfer impedance are shown.

Contents

This is a condensed translation of the original paper which appears in Japanese in the Journal of the Institute of Electrical Communication Engineers of Japan, No. 179, February, 1938.

I. The Porality of the Transfer Impedance

Now, let us consider the transfer impedance between the two pairs of terminals (1, 2) and (3, 4) in an arbitrary four-terminal passive network K, as shown in Fig. 1(i).

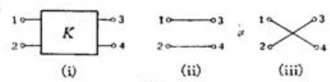

Fig. 1.

Simple partial paths contained in the network K are entirely arbitrary and each of them has an arbitrary time-impedance characteristic respectively. The connecting figure of the four points 1, 2, 3 and 4 in the network K, therefore, changes variously from time to time. If the total number of the connecting forms thus produced is denoted by N, we have the following relation:

$$N = \sum_{i=0}^{i=6} {}_6C_i = 64$$

Among these 64 connecting forms, however, the connecting forms, in case of which the energy can be freely passed through from a pair of terminals to the other pair, are only those two shown in Fig. 1 (ii) and (iii).

The transmission characteristic of the network K in the case of Fig. 1 (ii) has an opposite polarity with regard to that in the case of Fig. 1 (iii). In this paper, the phase constant of the transfer impedance in each case of Fig. 1 (ii) and (iii) is taken to be 0 and π respectively.

Regarding the transfer impedance, consequently, its absolute values are limitted to only two values, 0 and ∞, and its phase constants to only two values, 0 and π.

Comparing these facts with the propagation constant, $P=A+jB$, of the four-terminal passive network in the transmission circuit, following correspondencies are found:

	in the transmission circuit	in the relay circuit
independent variable	frequency	time
attenuation constant	A	0 or ∞
phase constant	B	0 or π

In these respects exists the peculiarity of the properties of the relay circuit.

Now, in order to develop a theory regarding the transfer impedance, we take the following notations:

$Z_t(0)=$ a positive transfer impedance, that is, a transfer impedance whose value is considered to be 0 only in the case of the connecting form shown in Fig. 1 (ii) and to be ∞ in all other cases

$Z_t(\pi)=$ a negative transfer impedance, that is, a transfer impedance whose value is considered to be 0 only in the case of the connecting form shown in Fig. 1 (iii) and to be ∞ in all other cases

$Z_t=$ an absolute transfer impedance, that is, a transfer impedance whose value is considered to be 0 only in the cases of the connecting form shown in Fig. 1 (ii) or (iii) and to be ∞ in all other cases

The reason why $Z_t(0)$ and $Z_t(\pi)$ are studied here is because it serves not only to the clarification of the physical meaning of Z_t but also to the treaties of acting phenomena in case, for an example, a source and a polarized relay are respectively connected to each side of a four-terminal passive network.

II. Method of obtaining Positive and Negative Transfer Impedances

Let us first consider the expressional form of $Z_t(0)$ referring to Fig. 1. From the definition of $Z_t(0)$ given in the previous section we know that the value of $Z_t(0)$ is zero only when both impedances Z_{13} (which means the two-point acting impedance viewed from the terminals 1 and 3 in the network K) and Z_{24} present zero value and impedances, Z_{12}, Z_{14}, Z_{23} and Z_{34}, all present infinite value, and is infinity in all other cases.

Accordingly, $Z_t(0)$ can be expressed as follows:

$$Z_t(0)=Z_{13}+Z_{24}+\overline{Z}_{12}+\overline{Z}_{14}+\overline{Z}_{23}+\overline{Z}_{34} \quad \cdots\cdots(1)$$

In the same manner, $Z_t(\pi)$ can be expressed by the followings:

$$Z_t(\pi)=Z_{14}+Z_{23}+\overline{Z}_{12}+\overline{Z}_{13}+\overline{Z}_{24}+\overline{Z}_{34} \quad \cdots\cdots(2)$$

Between the various two-point acting impedances of a four-terminal passive network, however, it hold true the following relations:

$$\left.\begin{aligned} &Z_{13}+Z_{24}+\overline{Z}_{12}=Z_{13}+Z_{24}+\overline{Z}_{14} \\ &=Z_{13}+Z_{24}+\overline{Z}_{23}=Z_{13}+Z_{24}+\overline{Z}_{34} \end{aligned}\right\} \cdots\cdots(3)$$

By applying the operation according to the theory of set to relations (3), we obtain the next relations:

$$\left.\begin{aligned} &Z_{13}+Z_{24}+\overline{Z}_{12}+\overline{Z}_{14}+\overline{Z}_{23}+\overline{Z}_{34} \\ &=Z_{13}+Z_{24}+\overline{Z}_{12}= \cdots\cdots \\ &=Z_{13}+Z_{24}+\overline{Z}_{12}+\overline{Z}_{14}= \cdots\cdots \\ &=Z_{13}+Z_{24}+\overline{Z}_{12}+\overline{Z}_{14}+\overline{Z}_{23}= \cdots \\ &=Z_{13}+Z_{24}+\overline{Z}_{12}\cdot\overline{Z}_{14}= \cdots\cdots \\ &=Z_{13}+Z_{24}+\overline{Z}_{12}\cdot\overline{Z}_{14}\cdot\overline{Z}_{23}= \cdots\cdots \\ &=Z_{13}+Z_{24}+\overline{Z}_{12}\cdot\overline{Z}_{14}\cdot\overline{Z}_{23}\cdot\overline{Z}_{34}= \cdots \end{aligned}\right\} \cdots\cdots(4)$$

Hence, from the expressions (1) and (4), we get various expressional froms of $Z_t(0)$ as follows:

$$\begin{aligned} Z_t(0)&=Z_{13}+Z_{24}+\overline{Z_{jk}} \\ &= \quad ,, \quad +\textstyle\sum_2\overline{Z_{jk}} \\ &= \quad ,, \quad +\textstyle\sum_3\overline{Z_{jk}} \end{aligned}$$

$$\left.\begin{aligned} &= \;\;,, \;\; +\textstyle\sum_4 \overline{Z_{jk}} \\ &= \;\;,, \;\; +\Pi_2 \overline{Z_{jk}} \\ &= \;\;,, \;\; +\Pi_3 \overline{Z_{jk}} \\ &= \;\;,, \;\; +\Pi_4 \overline{Z_{jk}} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots(5)$$

Z_{jk} contained in the above expression indicates any one of four kinds of impedances which remain by excluding Z_{13} and Z_{24} from six kinds of two-point acting impedances of a four-terminal passive network.

In the same manner the expressional form of $Z_t(\pi)$ is obtained as follows:

$$\left.\begin{aligned} Z_t(\pi) &= Z_{14} + Z_{23} + \overline{Z_{lm}} \\ &= \;,, \; +\textstyle\sum_2 \; \overline{Z_{lm}} \\ &= \;,, \; +\textstyle\sum_3 \; \overline{Z_{lm}} \\ &= \;,, \; +\textstyle\sum_4 \; \overline{Z_{lm}} \\ &= \;,, \; +\Pi_2 \; \overline{Z_{lm}} \\ &= \;,, \; +\Pi_3 \; \overline{Z_{lm}} \\ &= \;,, \; +\Pi_4 \; \overline{Z_{lm}} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots(6)$$

Where Z_{lm} indicates any one of four kinds of impedances which remain by excluding Z_{14} and Z_{23} from six kinds of two-point impedances of a four-terminal passive network.

By the expressions (5) and (6), we have now been able to express the positive and negative transfer impedances, $Z_t(0)$ and $Z_t(\pi)$, in terms of two-point acting impedances. It may be generally declared that the positive and negative transfer impedances with respect to any arbitrary two pairs of terminals in a four-terminal passive network can be obtained if the six kinds of two-point impedances are known.

Next, regarding the general basic type of the four-terminal passive network shown in Fig. 2, $Z_t(0)$ and $Z_t(\pi)$ are given as follows:

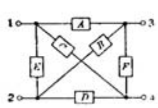

Fig. 2.

$$Z_t(0) = A + \overline{B} + \overline{C} + D + \overline{E} + \overline{F} \quad \cdots\cdots\cdots(7)$$

$$Z_t(\pi) = \overline{A} + B + C + \overline{D} + \overline{E} + \overline{F} \quad \cdots\cdots\cdots(8)$$

From the above relations or from general expressions (5) and (6) it may be easily understood that the reciprocity theorem holds true with regard to either the positive or negative transfer impedance.

Further, regarding the sum of $Z_t(0)$ and $Z_t(\pi)$ for the same two pairs of terminals of a four-terminal network, the next relation holds true.

$$Z_t(0) + Z_t(\pi) = p \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

III. The Relationship between the Positive and Negative Transfer Impedances and the Absolute Transfer Impedance—a Proof of $Z_t = Z_s + \overline{Z}_p$

Let us consider here what relation may exist between the positive-and negative transfer impedances which have been introduced in this paper and the absolute transfer impedance described in my previous paper. This problem can be solved in the following manner according to the Theory of Set. Considering the definitions of $Z_t(0)$, $Z_t(\pi)$ and Z_t given in Section II, it is clear that the impedance value of Z_t becomes zero in time regions in which either one of $Z_t(0)$ $Z_t(\pi)$ presents zero value, and becomes infinite in time regions in which the both present infinite value. Hence, according to the notion of the set, Z_t can be considered a common Set of $Z_t(0)$ and $Z_t(\pi)$, and the following relation is obtained:

$$\boxed{Z_t = Z_t(0).\, Z_t(\pi)} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

Now, Z_t will be obtained with regard to the general basic type of four-terminal networks as shown in Fig. 2.

Substituting the expressions of $Z_t(0)$ and $Z_t(\pi)$ respectively given by (7) and (8) into the equation (10), we have the following expression:

$$Z_t = (A + D)(B + C) + (\overline{A} + \overline{D})(\overline{B} + \overline{C}) + \overline{E} + \overline{F} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(11)$$

The equation (10) shows that Z_t can be expressed in a product form of $Z_t(0)$ and $Z_t(\pi)$ each of which is respectively reciprocal with respect to the transmitting direction of the energy. Z_t must, therefore, be also reciprocal. Hence, we can hold:

Regarding the energy transmission characteristics of a four-terminal passive network in the relay circuit, the Reciprocity Theorem can hold always true whether the porality is taken into consideration or not.

Next, let us derive out another expressional form of Z_t from the equation (10) and the properties of the two-point impedances. By substituting expressions (5) and (6) into the equation (10), we have

$$Z_t=(Z_{13}+Z_{24}+\overline{Z}_{12}+\overline{Z}_{34})$$
$$\times(Z_{14}+Z_{23}+\overline{Z}_{12}+\overline{Z}_{34}$$
$$=(Z_{13}+Z_{24})(Z_{14}+Z_{23})+\overline{Z}_{12}+\overline{Z}_{34} \quad \cdots(12)$$

According to relations between various two-point impedances of a four-terminal network, equation (12) can be transformed into the following:

$$Z_t=Z_{12}(\overline{3,4})+Z_{34}(\overline{1,2})+\overline{Z}_{12}+\overline{Z}_{34} \quad \cdots(13)$$

Where, $Z_{12}(\overline{3,4})$ indicates an impedance viewed from a pair of terminals 1 and 2 in the case of the other pair of terminals 3 and 4 being short-circuited.

On the other hand, we know that the following relation holds true in an arbitrary four-terminal passive network:

$$Z_{12}(\overline{3,4})+\overline{Z}_{12}=\overline{Z}_{34}(\overline{1,2})+\overline{Z}_{34} \quad \cdots\cdots\cdots(14)$$

By substituting the relation (14) into the relation (13) we have

$$\boxed{\begin{aligned} Z_t&=Z_{12}(\overline{3,4})+\overline{Z}_{12}\\ &=Z_{34}(\overline{1,2})+\overline{Z}_{34}\end{aligned}} \quad \cdots\cdots\cdots\cdots\cdots\cdots(15)$$

The above relation is nothing but the following relation given in my previous paper:

$$Z_t=Z_s+\overline{Z}_p$$

This result shows that the relation, $Z_t=Z_s+\overline{Z}_p$, has been analytically proven by the theory of the two-point impedance and the theory of the transfer impedance in which the polarity is taken into consideration.

IV. Mutual Relations of Various Transfer Impedances in the Case of Several Four-Terminal Passive Networks being Connected in Cascade

Now, let us consider about the case of several four-terminal passive networks being connected in cascade and obtain the relationship between various transfer impedances of the composite four-terminal network and those of its component four-terminal networks.

(A) The Case when two Four-Terminal Networks are Connected in Cascade

Let us consider the transfer impedance of a composite four-terminal network K (as shown in Fig. 3) composed of two arbitrary four-terminal networks, K_1 and K_2, connected in cascade.

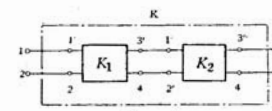

Fig. 3.

Let the transfer impedances for K_1, K_2 and K be denoted by Z_t', $Z_t'(0)$, $Z_t'(\pi)$, Z_t'', $Z_t''(0)$, $Z_t''(\pi)$ and Z_t, $Z_t(0)$, $Z_t(\pi)$, respectively.

The connecting form between the four terminals 1, 2, 3 and 4 of the composite four-terminal network K changes variously as simple partial paths involved in the component four terminal networks K_1 and K_2 present various time-impedance characteristics.

The total number of connecting forms which may happen is

$64\times64=4096$.

However, the cases when the terminals 1,3 and 2,4 are respectively connected, that is, the cases when the value of $Z_t(0)$ becomes zero are restricted to the cases when two connecting forms, as shown in Fig. 4 (a) and (b), are presented. The cases when terminals 1,4 and 2,3 are respectively connected, in other words, the cases when the value of $Z_t(\pi)$ becomes zero are restricted to the cases when two connecting forms, as shown in Fig. 4 (c) and (d), are presented.

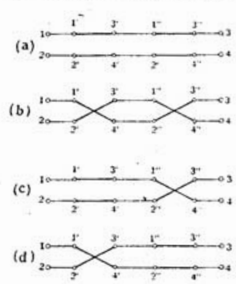

Fig. 4.

Now, let us consider first about $Z_t(0)$.

An impedance which becomes zero only when the connecting form (a) is present, and becomes infinity when the other connecting forms are present, may be expressed by $Z_t'(0)+Z_t''(0)$.

Likewise, an impedance which becomes zero only when the connecting form (b) is present, and becomes infinity when the other connecting forms are present, may be expressed by $Z_t'(\pi)+Z_t''(\pi)$. According to the relation (9), there can be no such a case when both of $\{Z_t'(0)+Z_t''(0)\}$ and $\{Z_t'(\pi)+Z_t''(\pi)\}$ become zero simultaneously. $Z_t(0)$ becomes always zero when either one of $\{Z_t'(0)+Z_t''(0)\}$ and $\{Z_t'(\pi)+Z_t''(\pi)\}$ presents zero value, and becomes infinity otherwise.

Hence, according to the Theory of Set, $Z_t(0)$ may be regarded as the common set of $\{Z_t'(0)+Z_t''(0)\}$ and $\{Z_t'(\pi)+Z_t''(\pi)\}$, and we obtain the following relation:

$$Z_t(0)=\{Z_t'(0)+Z_t''(0)\}\{Z_t'(\pi)+Z_t''(\pi)\} \qquad \cdots\cdots(16)$$

Next, as regard to $Z_t(\pi)$, it can be expressed in the same manner as follows:

$$Z_t(\pi)=\{Z_t'(0)+Z_t''(\pi)\}\{Z_t'(\pi)+Z_t''(0)\} \qquad \cdots\cdots(17)$$

Transforming the equations (16) and (17) by Operations according to the Theory of Set and applying the relation (10) to them, we have

$$Z_t(0)=Z_t'+Z_t''+Z_t'(0).Z_t''(\pi)+Z_t'(\pi).Z_t''(0) \qquad \cdots\cdots(18)$$

$$Z_t(\pi)=Z_t'+Z_t''+Z_t'(0).Z_t''(0)+Z_t'(\pi).Z_t''(\pi) \qquad \cdots\cdots(19)$$

Now, let us find out Z_t. This is obtained either from relations (10), (16) and (17) or from relations (10) (18) and (19) as follows:

$$\begin{aligned} Z_t &= Z_t(0).Z_t(\pi) \\ &= \{Z_t'+Z_t''+Z_t'(0).Z_t''(\pi)+Z_t'(\pi).Z_t''(0)\} \\ &\quad \times\{Z_t'+Z_t''+Z_t'(0).Z_t''(0)+Z_t'(\pi).Z_t''(\pi)\} \\ &= Z_t'+Z_t''+Z_t'(0).Z_t''+Z_t'.Z_t''(0) \\ &\quad +Z_t'.Z_t''(\pi)+Z_t'(\pi).Z_t'' \\ &= Z_t'+Z_t'' \end{aligned}$$

Hence, Z_t can be expressed in the following simple form:

$$\boxed{Z_t=Z_t'+Z_t''} \qquad \cdots\cdots(20)$$

If special relations exist between the transfer impedances of the two component four-terminal networks, several interresting results may be derived out from the above expressions. Some of them, as examples, are shown below:

(i) The Case when $Z_t'(0)=Z_t''(0)$ and $Z_t'(\pi)=Z_t''(\pi)$

In this case we have

$Z_t(0)=Z_t'=Z_t''$,

$Z_t(\pi)=p$ and

$Z_t=Z_t'=Z_t''$.

The negative transfer impedance is always infinite, and it needs to consider only the positive one.

(ii) The Case when $Z_t'(0)=Z_t''(\pi)$ and $Z_t'(\pi)=Z_t''(0)$

In this case we have

$Z_t(0)=p$,

$Z_t(\pi)=Z_t'=Z_t''$ and

$Z_t \; =Z_t'=Z_t''$.

The positive transfer impedance is always infinite, and it needs to consider only the negative one.

For instance, in case the same two four-terminal networks are connected in cascade form as shown in Fig. 5, the absolute transfer impedance of the composite network is the same as that of its component network, while, as regard to the polarity, it needs to consider only the negative one.

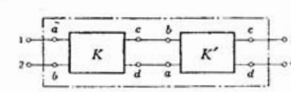

Fig. 5.

(iii) Cascade connection of the same two four-terminal passive networks whose positive and negative transfer impedances are mutually in the double-inverse relation.

When two component networks are connected in a form mentioned in the previous example (i), we have

$Z_t(0)=s$, $Z_t(\pi)=p$ and $Z_t=s$.

In the case of the two networks being connected in a form mentioned in the previous example (ii) we have

$Z_t(0)=p,\ Z_t(\pi)=s$ and $Z_t=s$.

In any case, the absolute transfer impedance is always zero and the polarity is either positive or negative.

This kind of composite four-terminal network does not change the wave form of energy to be transmitted but controls the polarity, if it exists, of the wave.

Simple examples are shown in Fig. 6. The component four-terminal network is a lattice type one and the acting impedance of its series element is in double-inverse relation with that of its shunt element. The upper composite network corresponds to the case (i) and the lower to the case (ii).

Intermediate figures show the wave forms at various points in the composite network.

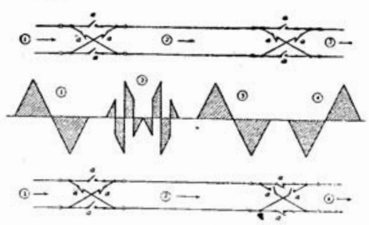

Fig. 6.

As for the ring modulator circuit using metal rectifiers (as shown in Fig. 7), its function, when considered ideal, may be explained by the function of one component four-terminal network shown in Fig. 6.

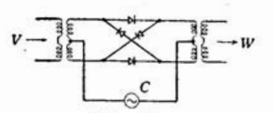

Fig. 7.

Metal rectifiers, when properly connected, can play simultaneously two roles, one as the acting element and the other as the contact of the so-called mechanical relay, and may be considered, in such cases, a kind of static relay.

This is the reason why metal rectifiers are being taken into transmission circuits instead of the magnetic relays.

(B) The Case when Three Four-Terminal Passive Networks are Connected in Cascade

Now let us find out the transfer impedance of a composite network composed of three arbitrary four-terminal passive networks connected in cascade as shown in Fig. 8.

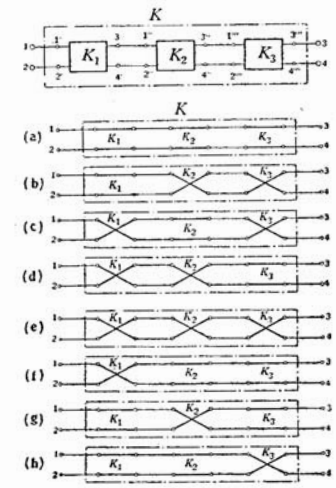

Fig. 8.

$Z_t(0)$ of the composite network K shown at the top in Fig. 8 becomes zero when any one of four connecting forms a, b, c and d in Fig. 8 is present and becomes infinity in case of all other connecting forms than those four. According to the property expressed by the relation (9), those four connecting forms do not occur simultaneously. Hence, $Z_t(0)$ can be expressed as follows:

$$\begin{aligned} Z_t(0)=&\{Z_t'(0)+Z_t''(0)+Z_t'''(0)\}\\ &\times\{Z_t'(0)+Z_t''(\pi)+Z_t'''(\pi)\}\\ &\times\{Z_t'(\pi)+Z_t''(0)+Z_t'''(\pi)\}\\ &\times\{Z_t'(\pi)+Z_t''(\pi)+Z_t'''(0)\} \quad \cdots\cdots\cdots(21)\end{aligned}$$

By transforming the expression (21), $Z_t(0)$ may also be expressed as follows:

$$\begin{aligned} Z_t(0)=&Z_t'+Z_t''+Z_t'''\\ &+Z_t'(0).Z_t''(0).Z_t'''(0)\\ &+Z_t'(0).Z_t''(\pi).Z_t'''(\pi)\\ &+Z_t'(\pi).Z_t''(0).Z_t'''(\pi)\\ &+Z_t'(\pi).Z_t''(\pi).Z_t'''(0) \quad \cdots\cdots\cdots\cdots\cdots(22)\end{aligned}$$

Next, as for $Z_t(\pi)$ of the composite network K, $Z_t(\pi)$ becomes zero when any one of the four connecting forms e, f, g, and h shown in Fig. 8 is present, and becomes infinity in all other cases.

Hence, in the same manner as in case of $Z_t(0)$, $Z_t(\pi)$ can be expressed by the following expression:

$$Z_t(\pi)=\{Z_t'(\pi)+Z_t''(\pi)+Z_t'''(\pi)\}$$
$$\times\{Z_t'(\pi)+Z_t''(0)+Z_t'''(0)\}$$
$$\times\{Z_t'(0)+Z_t''(\pi)+Z_t'''(0)\}$$
$$\times\{Z_t'(0)+Z_t''(0)+Z_t'''(\pi)\} \quad \cdots\cdots\cdots(23)$$

By rewriting the expression (23) we have

$$Z_t(\pi)=Z_t'+Z_t''+Z_t'''$$
$$+Z_t'(\pi).Z_t''(\pi).Z_t'''(\pi)$$
$$+Z_t'(\pi).Z_t''(0).Z_t'''(0)$$
$$+Z_t'(0).Z_t''(\pi).Z_t'''(\pi)$$
$$+Z_t'(0)Z_t''(0)Z_t'''(\pi) \quad \cdots\cdots\cdots\cdots\cdots\cdots(24)$$

The absolute transfer impedance Z_t of the composite network K can be obtained by substituting (23) and (24) into the relation (10) as follows:

$$Z_t=Z_t'+Z_t''+Z_t''' \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(25)$$

From these expressions we can clearly understand the mutual relationship between the various transfer impedances of a composite four-terminal network and those of its component networks.

As regard to the case when more than three four-terminal networks are connected in cascade, the transfer impedances can be also obtained in the same manner as above, and it can be easily proven that the absolute transfer impedance of the composite network is always expressed by the total sum of those of its component networks.

V. Various Properties Regarding the Transfer Impedance

Let us here find out several properties regarding the transfer impedance according to the general theory developed in previous sections.

(i) The case of equivalent simple partial paths being connected in parallel with all branches of a four-terminal passive network.

Let us denote the impedance of parallel simple partial path by G and the impedances of a network (in whose original form parallel simple partial paths are included but are here eliminated) by impedance symbols indicated by $'$ while those of the original network by impedance symbols without $'$.

Then, according to the law of superposition regarding two-point impedance, we have the following relations:

$$Z_t(0)=Z_t'(0)+\overline{G}, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(26)$$

$$Z_t(\pi)=Z_t'(\pi)+\overline{G} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(27)$$

and

$$Z_t=Z_t'+\overline{G} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(28)$$

These relations are graphycally shown in Fig. 9 in which K is an original network and K' is a network derived from the original one by eliminating its common parallel simple partial paths G.

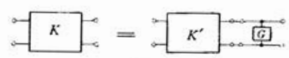

Fig. 9.

(ii) The case of equivalent simple partial paths being connected in series with all branches of a four-terminal passive network

With notations taken in similar manner to the previous case (i), transfer impedances are expressed as follows:

$$Z_t(0)=Z_t'(0)+G, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(29)$$

$$Z_t(\pi)=Z_t'(\pi)+G \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(30)$$

and

$$Z_t=Z_t'+G \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(31)$$

Where G is the impedance of series simple partial paths.

These relations are graphycally shown in Fig. 10 in which K is the original network and K' is a network derived from the original one by short-circuiting its common series simple partial paths G.

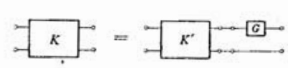

Fig. 10.

(iii) The transfer impedance of a lattice-type four-terminal passive network

A lattice-type four-terminal passive network is shown in Fig. 11, in which simple partial paths A, B, C and D are entirely arbitrary.

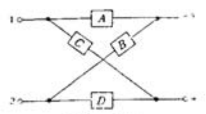

Fig. 11.

The transfer impedances of this network can be expressed as follows:

$$\left.\begin{aligned} Z_t(0) &= A+\overline{B}+\overline{C}+D \\ Z_t(\pi) &= \overline{A}+B+C+\overline{D} \\ Z_t &= (A+D)(B+C)+(\overline{A}+\overline{D})(\overline{B}+\overline{C}) \end{aligned}\right\} \cdots\cdots(32)$$

From expressions of (32) it will be found out the following various properties.

If the positions of series elements and shunt elements are mutually interchanged, the absolute transfer impedance remains unchanged while the positive and negative transfer impedances are changed to the negative and positive transfer impedances of the original network respectively.

The same results as above are also obtained if each one of series and shunt elements is replaced by a respective impedance in double-inverse relation with it.

Next, a network being given, let us derive from this two networks, the one by replacing only the series elements by impedances in double-inverse relation with them and the other by replacing only the shunt elements by impedances in double-inverse relation with them. Then the absolute transfer impedances of the two derived networks are the same while the positive and negative transfer impedances of the one are equal to the negative and positive transfer impedances of the other respectively. It is very interesting that these facts mentioned above coincide with variations of transmission characteristics of a lattice-type four-terminal passive network in the transmission circuit when its elements are similarly changed as in this case.

(iv) Balancing Conditions in Bridge Circuit

Referring to Fig. 11, the fact that the energy is not at all transmitted between two pairs of terminals, (1, 2) and (3, 4), means that two equations, $Z_t(0)=p$ and $Z_t(\pi)=p$, hold true simultaneously.

Hence, the balancing conditions may be obtained, according to relations (32), by solving the following simultaneous equations:

$$\left.\begin{aligned} A+\overline{B}+\overline{C}+D &= p \\ \overline{A}+B+C+\overline{D} &= p \end{aligned}\right\} \cdots\cdots(33)$$

This simultaneous equations have several solutions, one of which, as an similar one to that in case of the transmission circuit, is shown as follows:

$$AD=BC \quad \cdots\cdots(34)'$$

Besides this solution, any one of conditions shown below in (34)″ and (34)‴ is also the solution.

$$A=B,\ A=C,\ D=B \text{ and } D=C \cdots(34)''$$

$$A=BC,\ D=BC,\ B=AD \text{ and } C=AD \quad \cdots\cdots(34)'''$$

PART I

EXPANSION THEOREM AND DESIGN OF TWO TERMINAL RELAY NETWORKS (PART I.)*

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I. Expansion Theorem for Two Terminal Networks

1. Derivation of Expansion Theorem

"The Principle of Superposition for Separating Impedance between Two Terminals"(1) was shown to be the basic principle for the symbolic representation of two terminal impedances; from which, however complex the configuration of the network may be, the two terminal impedance Z may be represented by its component impedances X_1, $X_2$$\cdots\cdots$, X_n and the symbols $+$ and $\times$. That is, in general, Z may be expressed as a function of X_1, X_2,$\cdots\cdots$, X_n and the mathematical operators $+$ and $\times$. In other words

$$X \equiv F(X_1, X_2 \cdots\cdots, X_n, +, \times)$$

Since the algebra as given by George Boole in his unique work(2) has been used throughout the following mathematical analysis of passive relay circuits, the cupolas $+$ and $\times$ may consequently be omitted and the function simplified to

$$Z \equiv F(X_1, X_2 \cdots\cdots, X_n)$$

Hence, the impedance Z of a two terminal network will be expressed as a function F of the impedance of each of its component two-terminal networks. These functions will be designated as the impedance functions.

It was shown in a previous paper(3) that the impedance between terminals 1, 2 of a composite two terminal network K, constructed from a number of component two terminal networks and having network X_1 connected to any other terminals 3, 4, may be represented as shown in Fig. 1.

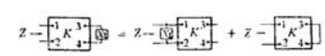

Fig. 1.

From inspection of Fig. 1, the general nature of the network suggests the following expansion:

$$\boxed{\begin{aligned}&F(X_1, X_2, \cdots\cdots, X_n)\\ &= X_1 \cdot F(p, X_2, \cdots\cdots, X_n) + F(s, X_2, \cdots\cdots, X_n)\end{aligned}} \quad \cdots\cdots\cdots\cdots(1)$$

where p and s are impedances having values of infinity and zero respectively, over the entire period under consideration.

In Eq. (1) if $X_1 \equiv p$

$$F(p_1, X_2, \cdots\cdots, X_n)$$
$$= F(p_1\ X_2, \cdots\cdots, X_n) + F(s, X_2, \cdots\cdots, X_n)$$

$$\therefore \quad \boxed{F(p, X_2, \cdots\cdots, X_n) \supset F(s, X_2, \cdots\cdots, X_n)} \quad \cdots\cdots\cdots\cdots(2)$$

In general, when the impedance of a composite two terminal network is expressed as an impedance function $F(X_1, X_2, \cdots\cdots, X_n)$, the expansion theorem as given in Eq. (1) can always be applied even though there are a number of component two-terminal networks having equal impedances.

Inversely, when the impedance of a composite two-terminal network is expressed as an impedance function $F(X_1, X_2, \cdots, X_n)$

* Jour. I.E.C.E., No. 206, May, 1940. (Condenced English translation of original Japanese paper)

the number of component two-terminal networks having equal impedances will not be restricted by the application of the expansion theorem as given in *Eq.* (1).

It will be shown later that this significant characteristic is the fundamental principle for two-terminal network designing.

From Eqs. (1) and (2), Eqs. (3) and (3′) may be derived.

$$F(X_1, X_2, \cdots\cdots, X_n) = X_1 \cdot F(p, X_2, \cdots, X_n) + \overline{X}_1 \cdot F(s, X_2, \cdots, X_n) \quad \cdots\cdots(3)$$

$$F(X_1, X_2, \cdots\cdots, X_n) = \{X_1 + F(s, X_2, \cdots\cdots, X_n)\} \cdot \{\overline{X}_1 + F(p, X_2, \cdots\cdots, X_n)\} \quad \cdots\cdots(3')$$

Following Eq. (3) and carrying out the expansion of function F with respect to all of its terms $X_1, X_2, \cdots\cdots, X_n$, Eq. (4) may be obtained.

$$\begin{aligned} F(X_1, X_2, X_3, \cdots\cdots, X_n) &= X_1X_2X_3\cdots\cdots X_nF(p, p, p, \cdots\cdots, p) \\ &+ \overline{X}_1X_2X_3\cdots\cdots X_nF(s, p, p, \cdots\cdots, p) + \cdots \\ &+ \overline{X}_1\overline{X}_2X_3 \cdots X_nF(s, s, p, \cdots, p) + \cdots \\ &\cdots\cdots \\ &+ \overline{X}_1\overline{X}_2\overline{X}_3\cdots\cdots\overline{X}_nF(s, s, s, \cdots\cdots, s) \end{aligned} \quad \cdots\cdots(4)$$

Similarly Eq. (4′) may be obtained by performing the operation as indicated in Eq. (3′).

$$\begin{aligned} F(X_1, X_2, \cdots\cdots, X_n) &= \{X_1 + X_2 + X_3 + \cdots + X_n + F(s, s, s, \cdots, s)\} \\ &\cdot \{\overline{X}_1 + X_2 + X_3 + \cdots X_n + F(p, s, s, \cdots, s)\} \cdots \\ &\cdot \{\overline{X}_1 + \overline{X}_2 + X_3 + \cdots + X_n + F(p, p, s, \cdots, s)\} \\ &\cdots\cdots \\ &\cdot \overline{X}_1 + \overline{X}_2 + \overline{X}_3 + \cdots + \overline{X}_n + F(p, p, p, \cdots, p)\} \end{aligned} \quad \cdots\cdots(4')$$

It is significant to note that the F ( ) in the right-hand side of Eqs. (4) and (4′) has a value of either p or s and represents a coefficient. The general expansion theorems as given in Eqs. (4) and (4′) are mutually of inverse forms. Since the same impedance function takes such distinctly opposite forms, it can be said that there exist two entirely opposite methods of procedure in designing two-terminal networks.

By substituting $p \equiv 1$ and $s \equiv 0$ for the values of p and s in Eqs. (3) and (4), these equations coincide with those given by George Boole[2] in his "Algebra of Logic," but unfortunately in his work the derivation of Eq. (3) was not given and Eq. (4) was based on the assumption of Eq. (3). Later, however, Ernst Schroder derived the solution of Eq. (3) diagramically[4]. From the mathematical standpoint the form of the equations as given by George Boole can be applied directly in this work. However, for the a forementioned reason, the writer endeavoured to clear the ambigiuty as to the derivation of the expansion theorems.

2. Characteristics of the Expansion Theorems

From analysis of the expansion theorems as given in Eqs. (4) and (4′) with respect to some of its operative character, the following convenient characteristics were found to exist:

(*i*) *Sum of Two Impedance Functions with the Same Variable*

$$\begin{aligned} &F_1(X_1, X_2, X_3, \cdots\cdots, X_n) + F_2(X_1, X_2, \cdots\cdots, X_n) \\ &= X_1X_2X_3\cdots X_n\{F_1(p,p,p,\cdots,p) + F_2(p,p,p,\cdots,p)\} \\ &+ \overline{X}_1X_2X_3\cdots X_1\{F_1(s,p,p,\cdots,p) + F_2(s,p,p,\cdots,p)\} + \cdots\cdots \\ &+ \overline{X}_1\overline{X}_2X_3\cdots X_n\{F_1(s,s,p,\cdots,p) + F_2(s,s,p,\cdots,p)\} + \cdots\cdots \\ &\cdots\cdots \\ &+ \overline{X}_1\overline{X}_2\overline{X}_3\cdots\overline{X}_n\{F_1(s,s,s,\cdots,s) + F_2(s,s,s,\cdots,s)\} \end{aligned} \quad \cdots\cdots(5)$$

$$\begin{aligned} &F_1(X_1, X_2, X_3, \cdots, X_n) + F_2(X_1, X_2, X_3, \cdots X_n) \\ &= \{X_1 + X_2 + X_3 + \cdots + X_n + F_1(s, s, s, \cdots, s) + F_2(s, s, s, \cdots, s)\} \\ &\cdot \{\overline{X}_1 + X_2 + X_3 + \cdots + X_n + F_1(p, s, s, \cdots, s) + F_2(p, s, s, \cdots, s)\} \cdots\cdots \\ &\cdot \{\overline{X}_1 + \overline{X}_2 + X_3 + \cdots + X_n + F_1(p, p, s, \cdots, s) + F_2(p, p, s, \cdots, s)\} \cdots\cdots \\ &\cdots\cdots \\ &\cdot \overline{X}_1 + \overline{X}_2 + \overline{X}_3 + \cdots + \overline{X}_n + F_1(p, p, p, \cdots, p) + F_2(p, p, p, \cdots, p)\} \end{aligned} \quad \cdots\cdots(5')$$

Accordingly, in the case of summation the variable terms remain unchanged, and it becomes only necessary to add the corresponding coefficient terms in the two funtions.

(*ii*) *Product of Two Impedance Functions with the Same Variable*

$$F_1(X_1, X_2, X_3, \cdots X_n)\cdot F_2(X_1, X_2, X_3, \cdots, X_n)$$
$$=X_1X_2X_3\cdots X_n\{p,p,p,\cdots,p)\cdot F_2(p,p,p,\cdots,p)\}$$
$$+\bar{X}_1X_2X_3\cdots X_n\{F_1(s,p,p,\cdots,p)\cdot F_2(s,p,p,\cdots,p\} +\cdots\cdots$$
$$+\bar{X}_1\bar{X}_2X_3\cdots X_n\{F_1(s,s,p,\cdots,p)\cdot F_2(s,s,p,\cdots,p)\} +\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\bar{X}_1\bar{X}_2\bar{X}_3\cdots\bar{X}_n\{F_1(s,s,s,\cdots,s)\cdot F_2(s,s,s,\cdots,s)\} \cdots\cdots\cdots\cdots(6)$$

$$F_1(X_1, X_2, X_3, \cdots X_n)\cdot F_2(X_1, X_2, X_3, \cdots, X_n)$$
$$=\{X_1+X_2+X_3+\cdots X_n + F_1(s,s,s,\cdots,s)\cdot F_2(s,s,s,\cdots,s)\}$$
$$\cdot\{\bar{X}_1+X_2+X_3+\cdots+X_n + F_1(p,s,s,\cdots,s)+F_2(p,s,s,\cdots,s\}\cdots\cdots$$
$$\cdot\{\bar{X}_1+\bar{X}_2+X_3+\cdots+X_n + F_1(p,p,s,\cdots,s)+F_2(p,p,s,\cdots,s)\}\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdot\{\bar{X}_1+\bar{X}_2+\bar{X}_3+\cdots+\bar{X}_n + F_1(p,p,p,\cdots,p)+F_2(p,p,p,\cdots,p)\}\cdots(6')$$

Consequently, in the case of product, the variable terms remain unchanged, and it becomes only necessary to multiply the corresponding coefficient terms in the two functions.

(*iii*) *Inverse Transformation of Impedance Function*

$$F(X_1, X_2, X_3, \cdots\cdots, X_n)$$
$$=X_1X_2\cdots\cdots X_n\bar{F}(p,p,p,\cdots\cdots,p)$$
$$+\bar{X}_1X_2X_3\cdots\cdots X_n\bar{F}(s,p,p,\cdots\cdots,p)+\cdots\cdots$$
$$+\bar{X}_1\bar{X}_2X_3\cdots\cdots X_n\bar{F}(s,s,p,\cdots\cdots,p)+\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\bar{X}_1\bar{X}_2\bar{X}_3\cdots\cdots\bar{X}_n\bar{F}(s,s,s,\cdots\cdots,s) \cdots\cdots\cdots\cdots(7)$$

$$\bar{F}(X_1, X_2, X_3, \cdots\cdots, X_n)$$
$$=\{X_1+X_2+X_3+\cdots+X_n+\bar{F}(s,s,s,\cdots,s)\}$$
$$\cdot\{\bar{X}_1+X_2+X_3+\cdots+X_n+\bar{F}(p,s,s,\cdots,s)\}\cdot\cdots$$
$$\cdot\{\bar{X}_1+\bar{X}_2+X_3+\cdots+X_n+\bar{F}(p,p,s,\cdots,s)\}\cdot\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdot(\bar{X}_1+\bar{X}_2+\bar{X}_3+\cdots+\bar{X}_n+\bar{F}(p,p,p,\cdots,p)\} \cdots\cdots\cdots\cdots(7')$$

Hence, in the case of inverse transformation the variable terms remain unchanged, and it becomes necessary to perform inverse transformation only on cach of the coefficient terms.

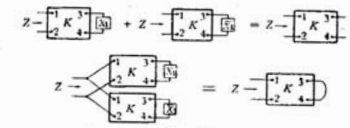

Fig. 2.

II. Investigation of Expansion Theorem from the Standpoint of Two-Terminal Network Designing

1. Fundamental Equations for Two-Terminal Network Designing

A pair of contact points is taken as the elementry units comprising the component two-terminal networks. When deliterating on the choice of a relay, it is sufficient to consider the two basic types of contacts, the M type and the B type of contacts. Hence, for the impedance of the component two-terminal networks, it is necessary to take only the M type and the B type of contact impedance for each relay.

The following notations will be used for the sake of convenience in the succeeding analysis.

Let

$A, B, C, \cdots\cdots, J$ = the given relays

$Z(A, B, C, \cdots\cdots, J)$ = the inpedance of the two-terminal network constructed from groups of contacts of relays $A, B, C, \cdots\cdots, J$.

($a, b, c, \cdots\cdots j$ = the M type contacts and their contact impedances of relays $A, B, C, \cdots\cdots, J$.)

$\bar{a}, \bar{b}, \bar{c}, \cdots\cdots, \bar{j}$ = the B type contacts and their contact impedance of relays $A, B, C, \cdots\cdots, J$.

Using the above notations and restating the previous expression, $Z(A, B, C, \cdots\cdots, J)$ in general becomes,

$$Z(A, B, C, \cdots\cdots, J) \equiv F(a,\bar{a},b,\bar{b},c,\bar{c},\cdots,j,\bar{j})$$

By expanding the right-hand side of the above equation using Eq. (4), the following is obtained:

$$F(a,\bar{a},b,\bar{b},c,\bar{c},\cdots,j,\bar{j})$$
$$=a\,b\,c\cdots jF(p.s,p,s,p,s,\cdots,p,s)$$
$$+a\,b\,c\cdots j\,F(s,p,p,s,p,s,\cdots,p,s)+\cdots\cdots$$
$$+a\,b\,c\cdots j\,F(s,p,s,p,p,s,\cdots,p,s)+\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\bar{a}\,\bar{b}\,\bar{c}\cdots\bar{j}\,F(s,p,s,p,s,p,\cdots,s,p)$$

In going through the above equation, it can be seen that a and $\bar{a}$, b and $\bar{b}$, ······ etc. have inverse relationships so that the complex form as given above may be reduced to the following simple expression:

$$\begin{aligned} & Z(A, B, C, \cdots, J) \equiv f(a, b, c, \cdots, j) \\ &= a\,b\,c \cdots j f(p, p, p, \cdots, p) \\ &\quad + \bar{a}\,b\,c \cdots j f(s, p, p, p, \cdots, p) + \cdots\cdots \\ &\quad + \bar{a}\,\bar{b}\,c \cdots j f(s, s, p, \cdots, p) + \cdots\cdots \\ &\quad \cdots\cdots\cdots\cdots\cdots\cdots \\ &\quad + \bar{a}\,\bar{b}\,\bar{c} \cdots \bar{j} f(s, s, s, \cdots, s) \end{aligned} \qquad \cdots\cdots(10)$$

It is important to note here that the function $f(a,\ b,\ c,\ \cdots\cdots,\ j)$ seemingly has terms for M type of contacts only, but it substantially contains terms for B type of contacts as well.

Similarly Eq. (10) may be obtained by using Eq. (4′).

$$\begin{aligned} & Z(A, B, C, \cdots, J) \equiv f(a, b, c, \cdots, j) \\ &= \{a + b + c + \cdots + j + f(s, s, s, \cdots, s)\} \\ &\quad \cdot \{\bar{a} + b + c + \cdots + j + f(p, s, s, \cdots, s)\} \cdots\cdots \\ &\quad \cdot \{\bar{a} + \bar{b} + c + \cdots + j + f(p, p, s, \cdots, s)\} \cdots\cdots \\ &\quad \cdots\cdots\cdots\cdots\cdots\cdots \\ &\quad \cdot \{\bar{a} + \bar{b} + \bar{c} + \cdots + \bar{j} + f(p, p, p, \cdots, p)\} \end{aligned} \qquad \cdots\cdots(10')$$

The designing of a two-terminal network, which follows, is based chiefly upon Eqs. (10) and (10′). Eqs. (10) and (10′) have forms identical with those of Eqs. (4) and (4′), which were given in Section I. Hence, the various characteristics of the general expansion theorems discussed in 2 of Section I can be applied directly here.

2. Types of Design Problem

The types of problems we are to face when designing two-terminal networks are several and many, but only those that can be solved by the application of the expansion theorems will be discussed in this article.

Design Problem (1)—Given μ number of relays $A,\ B,\ C,\ \cdots\cdots,\ J$ and to design a two-terminal network having the impedance s when any $\alpha (0 \leqq \alpha \leqq \mu)$ number of relays operate and having the impedance P for all other conditions.

Design Problem (2)—Given μ number of relays $A,\ B,\ C,\ \cdots\cdots,\ J$ and to design a two-terminal network having the impedance s when any α or β $(0 \leqq \alpha \leqq \mu,\ 0 \leqq \beta \leqq \mu,\ \alpha \neq \beta)$ number of relays operate and having the impedance p for all other conditions.

Design Problen (3)—Given μ number of relays $A,\ B,\ C,\ \cdots\cdots,\ J$ and to design a two-terminal network satisfying the prescribed impedance values for each operating combination but excluding those conditions already covered in Problems (1) and (2).

III. Discussions on Problems (1) and (2) from the Standpoint of Impedance Functions

Symbols used in the following discussion are as follows.

μ = the total number of relays $A,\ B,\ C,\ \cdots\cdots,\ J$.

$Z_\alpha(A,\ B,\ C,\ \cdots\cdots,\ J)$ = the two-terminal impedance of a network having the impedance s when any α out of μ number of relays operate, and having the impedance p for all other conditions.

$Z_{\alpha,\beta}(A, B, C, \cdots\cdots, J)$ = the two-terminal impedance of a network having the impedance s when any α or β out of μ number of relays operate, and having the impedance p for all other conditions.

1. Firstly, taking $Z_\alpha\ (A,\ B,\ C,\ \cdots\cdots,\ J)$, this corresponds to the general expansion equation (10) and (10′) if the value of the coefficient f ( ) is always s when ( ) contains α number of s's and if the value is p for all other conditions.

Secondly, analysing $Z_{\alpha,\beta}\ (A, B, C, \cdots, J)$ this corresponds to the general expansion equations (10) and (10′) if the value of the coefficient f ( ) is always s when ( ) contains α or β number of s's and if the value is p for all other conditions.

Taking into consideration the relationship between α and β in $Z_\alpha\ (A,\ B,\ C,\ \cdots\cdots,\ J)$ and $Z_{\alpha,\beta}\ (A,\ B,\ C,\ \cdots\cdots,\ J)$, the following fuctions can readily be derived from Eqs. (10) and (10′):

$$Z\,(A, B, \cdots, J) + Z_\beta(A, B, \cdots, J)$$

$= p$ ··(11)

$Z_\alpha(A, B, \cdots, J) \cdot Z_\beta(A, B, \cdots, J)$

$Z_{\alpha,\beta}(A, B, \cdots, J)$ ··(12)

$Z_{\alpha,\beta}(A, B, \cdots, J) + Z_{\beta,\gamma}(A, B, \cdots, J)$

$= Z_\beta(A, B, \cdots, J)$ ··(13)

$Z_{\alpha,\beta}(A, B, \cdots, J) \cdot Z_{\beta,\gamma}(A, B, \cdots, J)$

$= Z_{\alpha,\beta,\gamma}(A, B, \cdots, J)$ ································(14)

providing $\alpha \neq \beta \neq \gamma$.

2. Relationship between $\bar{Z}_\alpha(A, B, C, \cdots, J)$ and $Z_{0,1,2,\cdots,\alpha-1,\alpha+1,\cdots,\mu}(A, B, C, \cdots, J)$.—By combining Eqs. (7) and (7′) the following relationship may be found.

$$\bar{Z}_\alpha(A, B, C, \cdots, J) = Z_{0,1,2,\cdots,\alpha-1,\alpha+1,\cdots,\mu}(A, B, C, \ldots, J) \quad \cdots(15)$$

Furthermore, from Eqs. (12), (13) and (15), the inverse transformation relationship, as given below, can be deduced.

$$\bar{Z}_{\alpha,\beta}(A, B, C, \cdots, J) = Z_{0,1,2,\cdots,\alpha-1,\alpha+1,\cdots,\beta-1,\beta+1,\cdots,\mu}(A, B, C, \cdots, J) \quad \cdots(16)$$

3. Relationship between $Z_\alpha(A, B, C, \cdots, J)$ and $Z_{\mu-\alpha}(A, B, C, \cdots, J)$

In general the following characteristic exists. With the exception of the case when $\alpha = \mu - \alpha$, $Z_\alpha(A, B, C, \cdots, J)$ and $Z_{\mu-\alpha}(A, B, C, \cdots, J)$ always retain inverse contact relationship. Furthermore, since two networks are actually identical when $\alpha = \mu - \alpha$, $Z_{\mu/2}(A, B, C, \cdots, J)$ indicates a network containing characteristic of inverse contact. (In which case, the operating characteristics should not change by inverse contact transformation).

Also by comparison of $Z_{\alpha,\beta}(A, B, C, \cdots, J)$ and $Z_{(\mu-\alpha),(\mu-\beta)}(A, B, \cdots, J)$, the following relationship can easily be deduced from Eq. (12) and the previous conclusion. $Z_{\alpha,\beta}(A, B, C, \cdots, J)$ and $Z_{(\mu-\alpha),(\mu-\beta)}(A, B, C, \cdots, J)$ maintain an inverse contact relationship. However, these two cases become identical and indicate inverse contact networks when $\mu = \alpha + \beta$ or $\mu = 2\alpha = 2\beta$.

4. Relationship between $Z_{\mu-\alpha}(A, B, C, \ldots, J)$ and $Z_{0,1,2,\ldots,\alpha-1,\alpha+1,\ldots,\mu}(A, B, C, \ldots, J)$. From what is given above, it can be seen that $Z_\alpha(A, B, \ldots, J)$ and $Z_{0,1,2,\ldots,\alpha-1,\alpha+1,\ldots,\mu}(A, B, \ldots, J)$ have a double inverse relationship and $Z_\alpha(A, B, C, \cdots, J)$ and $Z_{\mu-\alpha}(A, B, \cdots, J)$ are of inverse contact forms. Hence by application of "The Fundamental Principle of Tetrahedron"[5] the following conclusion may be drawn:

$Z_{\mu-\alpha}(A, B, C, \cdots, J)$ and $Z_{0,1,2,\ldots,\alpha-1,\alpha+1,\ldots,\mu}(A, B, C, \cdots, J)$ have an inverse connection relationship.

Similarly, if both α and β are taken in place of α, $Z_{(\mu-\alpha),(\alpha-\beta)}(A, B, C, \cdots, J)$ and $Z_{0,1,2,\ldots,\alpha-1,\alpha+1,\ldots,\beta-1,\beta+1,\ldots,\mu}(A, B, C, \ldots, J)$ maintain an inverse connection relationship.

Also for this case, an inverse connection network can be found, e.g. a two-terminal network $Z_{1,2}(A, B, C)$, when $\mu = 3$.

5. Investigation of a Case When Number of Relays is Changed by One.—If the expansion of $Z_\alpha(A, B, \cdots, H, J)$ is performed in terms of the variable j, the following results.

$Z_\alpha(A, B, \cdots, H, J)$

$= j \cdot Z_\alpha(A, B, \cdots, H) + \bar{j} \cdot Z_{\alpha-1}(A, B, \cdots, H)$

If the above equation is rewritten in a form of a product.

$= \{Z_{\alpha-1}(A, B, \cdots, H)) + j\}$

$\cdot \{Z_\alpha(A, B, \cdots, H) + \bar{j}\}$

If ${}_\mu Z_\alpha$ is used to indicate the two-terminal impedance of a network having the impedance s when any α (out of the given μ number of relays) operate and having the impedance p for all other conditions, then the above equation becomes

$${}_{\mu+1}Z_\alpha = \{{}_\mu Z_{\alpha-1} + j\}\{{}_\mu Z_\alpha + \bar{j}\} \quad \cdots\cdots\cdots\cdots(17)$$

in which j and $\bar{j}$ are impedances of the M and the B type contacts when the number of relays is increased by one. Eq. (17) indicates that for Problem Type No. 1 a complex solution containing many relays can be easily deduced from a solution containing fewer relays.

The functional characteristics as given above is the basis of the analysis of problem type No. 1 and No. 2. The solution of the relationship of inverse contact, inverse conr nection and double inverse can easily be obtained when any one of the factor is obtained by application of the transformation method described in our previous paper[5].

IV. Designing Method for Problem No. 1

Eq. (17) in Section III indicates that the solution for the case of μ number of relays can be deduced directly from the solution of the case when $\mu=1$ since ${}_{\mu}Z_{\alpha}$ (when μ number of relays are given) can be obtained from ${}_{\mu-1}Z_{\alpha}$ and ${}_{\mu-1}Z_{\alpha-1}$ (when $\mu-1$ number of relays are given).

Firstly taking the case when $\mu=1$ from Eq. (10).

$$Z(A)\equiv f(a)=a\cdot f(p)+\bar{a}\cdot f(s)$$
$$\therefore\ Z_0(A)=\bar{a}$$
$$Z_1(A)=a$$

Also for $\mu=2$, from Eq. (17).

$$\left.\begin{aligned} Z_0(A,B)&=Z_0(A)+\bar{b}=\bar{a}+\bar{b}\\ Z_1(A,B)&=\{Z_0(A)+b\}\{Z_1(A)+\bar{b}\}\\ &=(\bar{a}+b)(a+\bar{b})\\ Z_2(A,B)&=\{Z_1(A)+b\}=a+b \end{aligned}\right\}\cdots(18)$$

Similarly for $\mu=3$

$$\left.\begin{aligned} Z_0(A,B,C)&(=Z_0(A,B)+\bar{c}=\bar{a}+\bar{b}+\bar{c}\\ Z_1(A,B,C)&=\{Z_0(A,B)+c\}\cdot\{Z_1(A,B)+\bar{c}\}\\ &=(\bar{a}+\bar{b}+c)\cdot\{(\bar{a}+b)(a+\bar{b})+\bar{c}\}\\ Z_2(A,B,C)&=\{Z_1(A,B,)+c\}\cdot\{Z_2(A,B)+\bar{c}\}\\ &=\{(\bar{a}+b)(a+\bar{b})+c\}\cdot\{a+b+\bar{c}\}\\ Z_3(A,B,C)&=\{Z_2(A,B)+c\}=a+b+c \end{aligned}\right\}\cdots\cdots(19)$$

The results obtained in Eqs. (18) and (19) can be seen in Fig. 3.

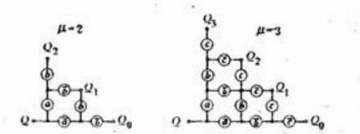

Fig. 3.

The two-terminal networks which are to be solved in Fig. 3 have one terminal at Q and the other terminal at Q_0, Q_1, Q_2 or Q_3 corresponding to Z_0, Z_1, Z_2 or Z_3, respectively.

On inspection of Fig. 3, the terminals, for example, for Z_2 (A, B, C) are Q and Q_2; the network, therefore contained within the boundaries, inclusive of the vertical and the horizontal lines drawn through the point Q_2, and the counter lines on the side of Q, represents the effective circuit and the remaining networks outside of this boundary, which were drawn for the purpose of representing the general case, can be neglected.

For this reason, the right hand part of Fig. 3 will be called "the fundamental network for problem No. 1 when the number of relays is three". Fig. 4 shows the connection diagram for the case when $\mu=3$.

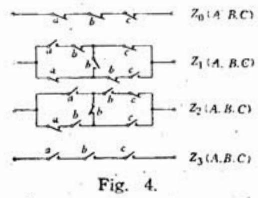

Fig. 4.

It can be seen that Z_1 and Z_2 are in inverse contact relation, which is a specific example of inverse contact relationship between Z_α and $Z_{\mu-\alpha}$, obtained in the previous section. Because of the similarity of the problem, it is possible to interchange a, b, c in Fig. 4.

In order to obtain the fundamental network when μ number of relays are given, like the case of $\mu=3$ being deduced from the case $\mu=2$, by applying inversely the principle of parallel resolution described in our former paper[2], the network can be successively transformed into forms of one higher order. This is shown in Fig. 5.

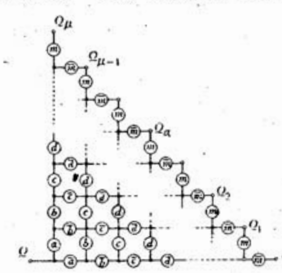

Fig. 5.

From the analysis of the fundamental network shown in Fig. 5, the designing procedure to be taken for Problem Type No. 1 are:

(i) Following Fig. 5, draw the fundamental network containing a given number of relays.

(ii) Let the two-terminal network having terminals at Q and Q_α represent Z_α which is to be solved.

(iii) Rewrite the figure, letting $a, b, c, \cdots\cdots$ represent the M type of contacts and $\bar{a}\ \bar{b}\ \bar{c}$ represent the B type of contacts.

(iv) Since the operating characteristic would not be changed by replacing a, b, c, replacement can be made freely, limited only by the number of contacts which the relays can accommodate. If the transfer types of contacts are desired, a replacement should be made.

(v) If ${}_\mu Z_\alpha$ can be obtained, ${}_\mu Z_{\mu-\alpha}$ can be obtained by a simple contact inversion.

V. Procedure for Problem Type No. 2

Similar to Problem 1, $Z_{\alpha,\beta}$ $(A, B, C, \cdots, J)$ can be solved by the method of parallel connection using Eq. (12) in 1 of Section III, which is

$Z_{\alpha,\beta} = Z_{\alpha\cdot\beta}$

For convenience, the general fundamental network for $Z_\alpha \cdot Z_\beta$ has been simplified in (1) of Fig. 6. By applying inversely the principle of parallel resolution, (1) of Fig. 7 can be transformed into its equivalent

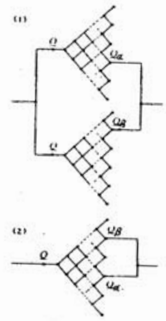

Fig. 6.

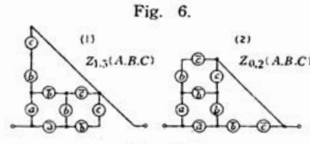

Fig. 7.

network as shown in (2). The networks to be obtained has one terminal at Q and has the other terminal at Q_α or Q_β which are short circuited.

Similarly the two-terminal network $Z_{\alpha,\beta}$, which is to be detained, has one terminal at point Q and has the other terminal at Q_α, Q_β or Q_γ which are short circuited. For example, two terminal networks satisfying $Z_{1,3}(A, B, C)$ and $Z_{0,2}$ (A, B, C) are shown in (1) and (2) of Fig. 7 respectively.

By examination it can be seen that the above figures have an inverse contact relationship; if (3) of Section 4 is referred to, it can be seen that this is only natural. By investigating the operating characteristics of these to networks, it is found that they have opposite impedances; which fact is clear from (2) of Section 4. In addition, from (4) of Section IV, it can be seen readily that the relationship of inverse contacts holds true also in this case.

VI. Procedure for Problem Type No. 3

In this section only those problems outside of Type No. 1 and Type No. 2 will be discussed. Consequently, the beauty of functional characteristics as found in the first two type problems will not be found. The design procedure for this type of problem is:

(i) Solve for the coefficient $f(\ \ )$ from the given conditions by using Eqs. (10) and (10′) given in Section III.

(ii) Express the equation of the desired two-terminal network in terms of the component impedance symbols.

(iii) Find a simpler two-terminal network by equivalent transformation. An example will be cited for clearness, as follows:

Given 4 relays A, B, C, D, obtain a two-terminal network containing the above relays and satisfying the following conditions when

A, B	operate and	C, D	release	$Z=s$
B, C	"	A, D	"	$Z=s$
A	"	B, C, D	"	$Z=s$
D	"	A, B, C	"	$Z=s$
for all other conditions				$Z=p$

Solution:

Expanding Z, using Eq. (10′),

$$Z(A,\ B,\ C,\ D)\equiv f(a,\ b,\ c,\ d)$$
$$=\{a+b+c+d+f(s,\ s,\ s,\ s,)\}$$
$$\cdot\{\bar{a}+b+c+d+f(p,\ s,\ s,\ s)\}\cdots\cdots$$
$$\cdot\{\bar{a}+\bar{b}+c+d+f(p,\ p,\ s,\ s)\}\cdots\cdots$$
$$\cdot\{\bar{a}+\bar{b}+\bar{c}+d+f(p,\ p,\ p,\ s)\}\cdots\cdots$$
$$\cdot\{\bar{a}+\bar{b}+\bar{c}+\bar{d}+f(p,\ p,\ p,\ p)\}$$

And analysing the coefficient terms above

$$f(s,\ s,\ p,\ p)=f(p,\ s,\ s,\ p)$$
$$=f(s,\ p,\ p,\ p)=f(p,\ p,\ p,\ s)=s$$

since for all other cases $f(\ \ )=p$. If these values are substituted in the above equation,

$$Z(A,\ B,\ C,\ D)$$
$$=(a+\bar{b}+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+d)$$
$$\cdot(\bar{a}+b+c+\bar{d})(a+b+\bar{c}+\bar{d})$$

Thus, by the above the composition of the desired two-terminal network has been determined. This result is shown in (1) of Fig. 8. This figure can be transformed into its equivalence, figure (2), by applying inversely the principle of parallel resolution.

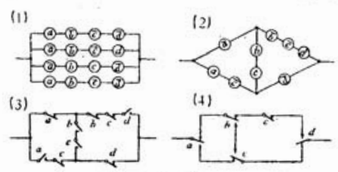

Fig. 8.

Fig. (3) is the contact diagram of figure (2) and if (3) is transformed using transfer cantact instead, figure (4) results. In these contact diagrams, the contact arms always operate in a clockwise direction.

The example given above covers all possible solutions which can be obtained by the equivalent transformation using (1) of Fig. 8 as the base.

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PART I

EXPANSION THEOREM AND DESIGN ON TWO TERMINAL RELAY NETWORKS (PART II)*

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VII. Problem Type No. 4

1. Conditions of the problem

Conditions for the fourth type of problem are "Given μ number of relays, to design a two terminal network having impedance s when any odd number (or even number) of relays operate and having impedance p when any even number (or odd number) of relays operate".

2. Notations

The following rules of notation will be used in deriving the impedance equations.

$X_1, X_2, \cdots\cdots, X_\mu$ = Given relays

$x_1, x_2, \cdots\cdots, x_\mu$ = Impedances of M type contacts of corresponding relays.

$\bar{x}_1, \bar{x}_2, \cdots\cdots, \bar{x}_\mu$ = Impedances of B type contacts of corresponding relays.

$\underset{\text{odd}}{Z}(X_1, X_2, \cdots\cdots, X_\mu)$ = Impedance of a two terminal network, containing μ number of relays, $X_1, X_2, \cdots\cdots, X_\mu$, having impedance s when any odd numbers of relays operate and having impedance p at all other conditions.

$\underset{\text{even}}{Z}(X_1, X_2, \cdots\cdots, X_\mu)$ = Impedance of a two terminal network containing μ number of relays, $X_1, X_2, \cdots\cdots, X_\mu$, having impedance s when any even number of relays operate and having impedance p at all other conditions.

Using these notations in Eq. (12) and Eq. (16) in Part I of this paper[1], following equations can be derived.

$$\left.\begin{aligned} \underset{\text{odd}}{Z}(X_1, X_2, \cdots, X_\mu) &= \prod_{n=0}^{n} Z_{2n+1}(X_1, X_2, \cdots, X_\mu) \\ \underset{\text{even}}{Z}(X_1, X_2, \cdots, X_\mu) &= \prod_{n=0}^{n} Z_{2n}(X_1, X_2, \cdots, X_\mu) \end{aligned}\right\} \cdots\cdots\cdots\cdots(20)$$

$$\underset{\text{odd}}{Z}(X_1, X_2, \cdots, X_\mu) = \overline{\underset{\text{even}}{Z}(X_1, X_2, \cdots, X_\mu)} \cdots(21)$$

The final value of n in Eq. (20) changes depending on μ being odd or even. Also Eq. (20) indicates that this problem can be treated under Problem Type No. 2 as one of its cases. Eq. (21) shows that these two terminal networks have the relationship of double inversion, therefore by analysing one the other relationship can easily be found. Using Eq. (10)[1],

$\underset{\text{odd}}{Z}(X_1, X_2, \cdots\cdots, X_\mu)$ becomes as follows:

$$\begin{aligned} &\underset{\text{odd}}{Z}(X_1, X_2, X_3, X_4, X_5, \cdots, X_\mu) \\ &= x_1\, x_2\, x_3\, x_4\, x_5 \cdots x_\mu \\ &\quad + \bar{x}_1\, \bar{x}_2\, x_3\, x_4\, x_5 \cdots x_\mu + \cdots\cdots\cdots \\ &\quad + \bar{x}_1\, \bar{x}_2\, \bar{x}_3\, \bar{x}_4\, x_5 \cdots x_\mu + \cdots\cdots\cdots \\ &\quad + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \cdots\cdots\cdots\cdots(22)$$

The form of Eq. (22) is awkward and does not lead to simple generalization, therefore at this point new operators used by B. A. Bernstein[2] in his analysis of Boolean algebra has been introduced. In his thesis[2], Complete Disjunction and its inverse were represented with symbols ○ and △ respectively. Here, however, it was arbitrarily decided to use, for clearity, the operational symbols ⊕ and ⊗. These operators were introduced into the impedance functions.

In connection with impedances $A_1, A_2, \cdots\cdots$, having any voluntary time character, the new operators ⊕ and ⊗ and the former

* Jour I. E. C. E., No. 209. August. (Condensed English translation of Original Japanese paper)

operators + and × have definite relationship as shown below.

$$A_1 \oplus A_2 = A_1 \times \bar{A}_2 + \bar{A}_1 \times A_2$$
$$= (A_1 + A_2) \times (\bar{A}_1 + \bar{A}_2) \cdots\cdots\cdots\cdots (23)$$
$$A_1 \otimes A_2 = A_1 \times A_2 + \bar{A}_1 \times \bar{A}_2$$
$$= (A_1 + \bar{A}_2) \times (\bar{A}_1 + A_2) \cdots\cdots\cdots\cdots (24)$$

In general commutative law and distributive law exist for operators ⊕ and ⊗.

The correlations between terms connected with operators ⊕ and ⊗ are as follows:

$$A_1 \oplus A_2 = \overline{(A_1 \otimes A_2)} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (25)$$
$$\left.\begin{aligned} A_1 \oplus \bar{A}_2 &= A_1 \otimes A_2 \\ A_1 \oplus A_2 &= A_1 \otimes \bar{A}_2 \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (26)$$

Moreover, for positive integer of n

$$A_1 \oplus A_2 \oplus \cdots\cdots \oplus A_{2n}$$
$$= \overline{(A_1 \otimes A_2 \otimes \cdots\cdots \otimes A_{2n})} \cdots\cdots\cdots\cdots\cdots (27)$$
$$A_1 \oplus A_2 \oplus \cdots\cdots \oplus A_{2n+1}$$
$$= A_1 \otimes A_2 \otimes \cdots\cdots \otimes A_{2n+1} \cdots\cdots\cdots\cdots (28)$$

From Eqs. (27) and (28) it can be seen that the relationship changes depending on the number of impedances, whether the number is odd or even.

By introducing the new operators, discussed above, Eq. (22) becomes a simple expression as given below:

$$\boxed{\begin{aligned} &\underset{\text{odd}}{Z}(X_1,\ X_2,\ X_3, \cdots\cdots,\ X_\mu) \\ &= x_1 \otimes x_2 \otimes x_3 \otimes \cdots\cdots,\ \otimes x_\mu \end{aligned}}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots (29)$$

Eq. (29) holds for all values of μ, irrespective of μ being odd or even.

$\underset{\text{even}}{Z}(Z_1,\ Z_2,\ Z_3 \cdots\cdots,\ Z_\mu)$ can be derived readily from Eq. (29), however, the form of the equation changes according to μ being odd or even.

When μ is even:

$$\boxed{\begin{aligned} &\underset{\text{even}}{Z}(X_1,\ X_2,\ X_3, \cdots\cdots,\ X_\mu) \\ &= x_1 \oplus x_2 \oplus x_3 \oplus \cdots\cdots \oplus x_\mu \end{aligned}}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots (30)$$

When μ is odd:

$$\boxed{\begin{aligned} &\underset{\text{even}}{Z}(X_1,\ X_2,\ X_3, \cdots\cdots,\ X_\mu) \\ &= x_1 \oplus x_2 \oplus x_3 \oplus \cdots\cdots \otimes\ x_\mu \end{aligned}}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots (31)$$

As shown in the above analysis the impedance of two terminal network satisfying Problem Type No. 4 has been simplified with the introduction of new operators ⊕ and ⊗.

3. Construction of Connection Diagram

The connection diagram of the desired two terminal network will be constructed by using the impedance function given in the preceeding section.

To begin with, take Fig. 9 and assume $\mu=2$.

This requirement can be fulfilled by selecting a two terminal network having a terminal at each side of the network shown in Fig. 9. Accordingly, if notation $W_{1,3}$ is used to represent the impedance of a two terminal network having its terminals at points 1 and 3, following equations can be written directly from analysis of Fig. 9.

$$W_{1,3} = (x_1 + x_2)(\bar{x}_1 + \bar{x}_2) = x_1 \oplus x_2$$
$$W_{1,4} = (x_1 + \bar{x}_2)(\bar{x}_1 + x_2) = x_1 \otimes x_2$$
$$W_{2,3} = (\bar{x}_1 + x_2)(x_1 + \bar{x}_2) = \bar{x}_1 \oplus x_2 = x_1 \otimes x_2$$
$$W_{2,4} = (\bar{x}_1 + \bar{x}_2)(x_1 + x_2) = \bar{x}_1 \otimes x_2 = x_1 \oplus x_2$$

Next, considers the case when $\mu=3$.

By performing the operation, following relationship existing between the two conditions can be found.

$$x_1 \oplus x_2 \oplus x_3 = (x_1 \oplus x_2) \oplus x_3$$
$$= (x_1 \oplus x_2) + x_3 \times \overline{(x_1 \oplus x_2)} + x_3$$
$$= (x_1 \oplus x_2) + x_3 \times (x_1 \otimes x_2) + \bar{x}_3$$
$$x_1 \otimes x_2 \otimes x_3 = (x_1 \otimes x_2) \otimes x_3$$
$$= (x_1 \otimes x_2) + \bar{x}_3 \times \overline{(x_1 \otimes x_2)} + x_3$$
$$= (x_1 \otimes x_2) + \bar{x}_3 \times (x_1 \oplus x_2) + x_3$$

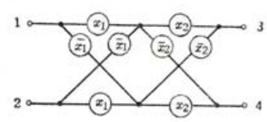

Fig. 9.

The construction as indicated in the above equation requires these two, two-terminal networks to be connected in parallel, moreover, two two-terminal networks $(x_1 \oplus x_2)$ and $(x_1 \otimes x_2)$ have one point in Fig. 9, as their common terminal, their other terminals being two other different points. The two terminal network obtained in the foregoing discussion has been indicated in solid line in Fig. 10, which has terminals at points 1 and 3. Also in the same diagram, if the dotted part of the network is added, the two terminal network having terminals at points 1 and 4 represents the double-inversion network. This becomes clear from Eq. 25.

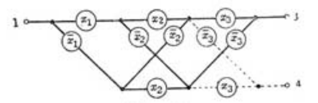

Fig. 10.

By following the procedure outlined above, the connection diagram of two terminal impedance which satisfies the right hand side of Eq. (29), namely $x_1 \otimes x_2 \otimes x_3 \otimes \cdots\cdots \otimes x_\mu$, will vary depending whether μ is odd or even.

the case when μ is odd ······Fig. 11 (i)
the case when μ is even······Fig. 11 (ii)

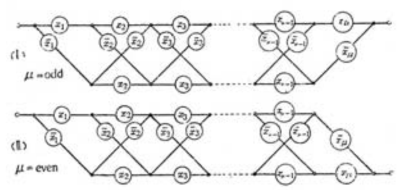

Fig. 11.

Nextly, consider the impedance $\underset{\text{even}}{Z}(X_1, X_2, X_3, \cdots\cdots, X_\mu)$.

This impedance can be realized by performing on the last section of the network shown in Fig. 11, the transformation as indicated in Fig. 10.

Now, assuming that the contacts x_k ($k=1, 2, \cdots\cdots, \mu$) and contacts $\bar{x}_k$'s to be connected in the horizontal and the diagonal branches, respectively, the required two terminal network having impedances $\underset{\text{odd}}{Z}(X_1, X_2, X_3, \cdots\cdots, X_\mu)$ and $\underset{\text{even}}{Z}(X_1, X_2, X_3, \cdots\cdots, X_\mu)$ can be expressed in general with Fig. 12.

The two terminal network which satisfies Problem Type No. 4 is shown in Fig. 12, in which the M type contacts of each relay comprise the series elements and the B type contacts comprise the shunt elements.

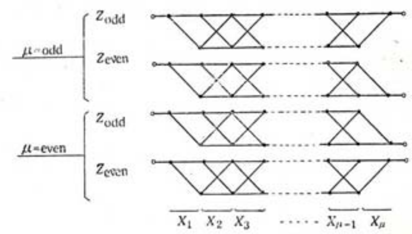

Fig. 12.

4. Practical Significance of Problem Type No. 4

The practical significance of this type of problem will be discussed. Since μ, the number of given relays, is always an integer, if the contact position of any one relay is altered, the total number of relays in operate position changes from odd to even or from even to odd. Therefore, the impedance of two terminal network satisfying this problem always changes from p to s or from s to p when the contact position of any one relay changes.

As an application of this operating characteristic controlling of a single electric light from any of the μ number of switches in series located at different points could be given. Fig. 13 shows one case of this application.

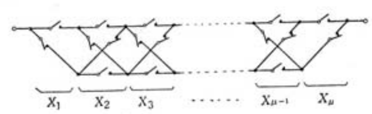

Fig. 13.

VIII. Problem Type No. 5

1. Conditions of the problem

The conditions for the fifth type of problem are:

"To design a two terminal network having impedance of specified time character, when number of relays and their corresponding time characters are given."

This problem can be handled as the problem of determination of the coefficients of the general expansion equation as discussed in the previous part of this paper(1). Moreover, following, two types of methods of determining the coefficients could be enumerated.

(1) the method adhereing strictly to operations of symbolic equations.

(2) the method using the time spectrum diagrams of impedances.

The first method has the advantage that only mathematical operations are required, however, complication is met in the simultaneous solution of symbolic, impedance equations. The second method requires construction of the diagrams, but has the advantage that the coefficients could be determined directly by inspection and that the possibility or impossibility of solution becomes evident at a glance.

2. Method using symbolic equations

The general procedure of this method is as follows:

i) Express the impedance of M type contact of each relay and the required two-terminal impedance in terms of symbolic equation using unit impedance. Also obtain the relationship between contact impedances.

ii) Write the general expansion equation of the desired impedance Z, and obtain the value of the coefficients excluding the terms in which combination of p and s are different from relationship of contact inpedance obtained in i).

iii) In the remaining terms, equate the impedance factors to their corresponding elements, p or s, within the parenthesis. Then obtain the value of unit impedances contained in the impedance factors. Substitute these values of unit impedances in the equation for Z expressed in terms of unit impedances.

An example will be cited to show concretely the design procedure.

Example:

Given 3 relays A, B, C whose time characters are as follows:

A……operates at T_1, releases at T_3
B…… 〃 〃 T_2, 〃 〃 T_4
C…… 〃 〃 T_5, 〃 〃 T_6

and to obtain a two terminal network having impedance Z as specified below:

$$Z=M_1+B_2\cdot M_5+B_6$$

Solution:

Firstly, if the impedance of M type contacts of each relay is expressed in terms of unit impedance,

then,

$$a=M_1+B_3, \quad b=M_2+B_4$$
$$c=M_5+B_6$$

The relationships existing between these impedances are

$$a\supset\bar{c}, \quad b\supset\bar{c}$$

If the coefficients which contradict the above relationship, namely $a=s$, $c=s$ or $b=s$, $c=s$, are omitted from the general expansion equation, it may be expressed as follows:

$$\begin{aligned}Z&\equiv f(a,b,c)\\&=abc\cdot f(p,p,p)+\bar{a}bc\cdot f(s,p,p)\\&\quad+a\bar{b}c\cdot f(p,s,p)+ab\bar{c}\cdot f(p,p,s)\\&\quad+\bar{a}\bar{b}c\cdot f(s,s,p)\end{aligned}$$

Therefore, it becomes necessary only to solve for the coefficients in the above equation. For example, the determination of $f(s,p,p)$ requires the solution of the simultaneous equation,

$$\left.\begin{aligned}a&=M_1+B_3=s\\b&=M_2+B_4=p\\c&=M_5+B_6=p\end{aligned}\right\}$$

by which

$$M_1=s,\ M_2=M_3=M_4=M_5=M_6=p$$
$$B_1=p,\ B_2=B_3=B_4=B_5=B_6=s$$

can be obtained, and substituting these values in the equation for Z, $Z=s$ may be obtained. Namely,

$$f(s,p,p)=s$$

After obtaining in this manner the values of coefficients and substituting these values in the expansion equation, Z becomes a function of a, b, c as shown below:

$$\begin{aligned}Z&=abc\cdot p+\bar{a}bc\cdot s+a\bar{b}c\cdot p\\&\quad+ab\bar{c}\cdot s+\bar{a}\bar{b}c\cdot p\\&=abc+a\bar{b}c+\bar{a}\bar{b}c\end{aligned}$$

And if the operation of Boole's algebra is applied to the equation, it becomes

$$Z=(a+\bar{b})c$$

or again using the relationship $\bar{b}\subset c$ and after transforming the right hand side of the equation, Z becomes

$$Z=ac+\bar{b}$$

The two terminal network specified by the right hand side of the above equation must necessarily have a configuration as given in Fig. 14 or a figure equivalent to it.

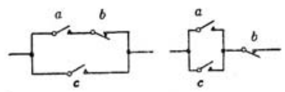

Fig. 14.

3. Method using time spectrum diagrams of impedances

The method using the time spectrum diagrams of impedances is outlined below:

i) Draw the time spectrum diagrams of the M type impedances of given relays and of the required two-terminal network.

ii) Divide the entire time period at points at which the relays change their contact character, and determine the coefficients of the expansion equation by inspecting the combination of the impedance values.

The design procedure using this method will be described.

Example :

Example problem stated in Section 2 will again be used.

Solution :

Fig. 15 shows the time spectrum diagrams of the impedances of the M type contacts a, b, c and impedance $Z=M_1+B_2M_5+B_6$ of the desired two terminal network.

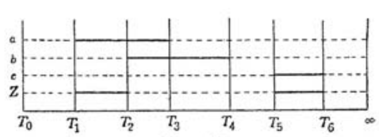

Fig. 15.

In the spectrum diagram, the solid line represents zero value and the dotted line represents infinite value. Time is taken on the horizontal axis. If the impedance values during each time interval are examined and compared following relationship may be found.

time interval $(T_0T_1)\cdots\cdots f(p, p, p)=p$
" " $(T_1T_2)\cdots\cdots f(s, p, p)=s$
" " $(T_2T_3)\cdots\cdots f(s, s, p)=p$
" " $(T_3T_4)\cdots\cdots f(p, s, p)=p$
" " $(T_4T_5)\cdots\cdots f(p, p, p)=p$
" " $(T_5T_6)\cdots\cdots f(p, p, s)=s$
" " $(T_6T\infty)\cdots f(p, p, p)=p$

No contradiction exists between the coefficients, therefore, solution of this problem is possible. (no solution being possible if there were contradictions).

From above the required two terminal impedance Z may be expressed by the following equation:

$$Z=abc+a\bar{b}c+\bar{a}\bar{b}c$$

This solution is same as one obtained in Section 2.

It may be seen from above that the time spectrum method of determining coefficients is more convenient and, as stated before, has the advantage that possibility or impossibility of solution can readily be determined. The author has found this method to be superior in practical applications.

IX. Conclusion

In the foregoing article the design theory for two terminal networks has been developed by application of the expansion theorem for impedance functions, and the general solution and the basic network configuration has been obtained for several problems. It may be mentioned that the problem of designing two terminal networks is not restricted to the five types mentioned in the foregoing discussions; other problems will be discussed at some other later date.

Acknowledgement :

The authors take pleasure in acknowledging their indebtedness to Dr. Y. Niwa and their associates for their valuable assistance in preparing this paper.

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中嶋，榛澤兩氏「繼電器回路に於けるインピーダンス函數の展開定理と二端子回路網の設計理論」に對する質疑及意見

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(電氣試驗所第二部)

上記論文に對する質疑及意見を二三述べさせて戴き度いと思ふ。

(1) 上記論文に於て著者は繼電器回路に對する美麗なる展開式を誘導された。小生は斯る展開式の一つの價値はその形式の美麗さにあるものと推察確信するが故に次の質疑を試み度い。

其の (1) 第 IV 節に於て Z_α, Z_β を作動素の何れか α 及び β 個が動作する場合インピーダンス値が s なる二端子インピーダンスとなして居る。著者は最初よりインピーダンス ∞ なる要素を以て集合の要素となしたるが故に飽く迄も此の態度を保持して Z_α, Z_β を作動素の何れか α 及び β 個が動作する時インピーダンスが ∞ となる二端子インピーダンスとなしたる方が最初の定義にも一致し結果が美麗になるのではないであらうか。即ち

$Z_\alpha(A,B,C \ldots\ldots J)$＝何れか α 個が動作する時インピーダンスが p 其他の場合にはインピーダンス s なる如き二端子回路網の二端子インピーダンス

$Z_{\alpha,\beta}(A,B,C, \quad J)$＝何れか α 個或は何れか β 個が動作する時インピーダンスp，其他の場合にはインピーダンス s なる如き二端子回路網の二端子インピーダンス

となしインピーダンスが p となるときを集合の基準に取れば

$$Z_\alpha(A,B,C\ldots)+Z_\beta(A,B,C\ldots J)=Z_{\alpha,\beta}(A,B,C\ldots J)$$

$$Z_\alpha(A,B,C\ldots J)\cdot Z_\beta(A,B,C\ldots J)=0 \text{ (空集合)}$$

$$Z_{\alpha,\beta}(A,B,C\ldots J)+Z_{\beta,\gamma}(A,B,C\ldots J) =Z_{\alpha,\beta,\gamma}(A,B,C\ldots J)$$

$$Z_{\alpha,\beta}(A,B,C\ldots J)\cdot Z_{\beta,\gamma}(A,B,C\ldots J) =Z_\beta(A,B,C,\ldots J)$$

であり，Z_α, Z_β 等に於ても＋符號は兩方の要素を含む集合，即ち $Z_\alpha+Z_\beta=Z_{\alpha,\beta}$, $Z_{\alpha,\beta}+Z_{\beta,\gamma}=Z_{\alpha,\beta,\gamma}$ と直ちに書き表され，又積符號は兩方の集合に共通な要素を含む集合となし $Z_\alpha\cdot Z_\beta=0$ $Z_{\alpha,\beta}$ $Z_{\beta,\gamma}=Z_\beta$ と書かれる。即ちインピーダンスに對しても集合の和及積の意義を一致せしめ又關係式も一層見易い美麗な結果となる。

次に最初の和及積の約束に戻り集合の要素としてインピーダンスが ∞ (開路) となる要素を取るべきか，インピーダンスが 0 (閉路) となる要素を取るべきかの問題である。何れでも差支ない筈であるが，實際今上の例で著者がインピーダンス 0 なる方に着目せられた樣に一般の人 (Piesch も此の一人) も無意識に此方の方を取り易いのではなからうか。之は理論の問題でなく一般人の考慮し易い傾向の問題である。インピーダンス 0 (閉路) を集合の要素とすれば直列にある接點及並列にある接點の表現形式が夫々積及和になるのは云ふ迄もないが，將來此の記號を廣く採用する上に於て大切なことであるから著者の御意見を承り符號の確立を期し度いと思ふ。

(2) 著者は，或る繼電器回路網インピーダンス變化が夫等繼電器の復舊及動作狀態の函數として與へられた場合，之を滿足する回路網の實現に關し，美麗且嚴密なる展開式を試みられた。

併し乍ら繼電器回路に於ける此種問題は具體的事實としてはさう複雜難解な問題でなく，却つて集合の思想と順列及組合せの思想とを以て素朴に取扱つた方が一般の人に了解され又印象的でもあるのではないかと思はれる。實は著者の解法の武器として採用せられたBoole代數學なるものも全く集合及順列組合せにより誘導せられたものであらうから結局同樣であらうが，平易に了解し得られる如く小生は敢て次の如き說明を推さうと思ふ。玆に繼電器組の動相 (假稱，動作形態) 及之に對する特有インピーダンス (假稱) なる觀念(と云ふより名稱) を導き入れた。

今　$X_1, X_2,\ldots, X_n=n$ 個の繼電器の名稱
　　$X_1, X_2,\ldots, X_n$ を以て同上繼電器の復舊狀態
　　$\overline{X}_1, \overline{X}_2,\ldots, \overline{X}_n$ を以て同上繼電器の動作狀態を表はす。

即ち X_i X_j X_m が動作狀態であり他の總ての繼電器が復舊狀態なる組合せを

$(X_1, X_2, X_3\ldots\overline{X}_i,\ldots\overline{X}_j,\ldots\overline{X}_m, X_n,)$ で表はす。

The first page of the paper
Akira Nakashima and Masao Hanzawa,
"The Answer to the above Question and Comments",
Journal of the Institute of Electrical Communication Engineers of Japan,
November 1941, 665-666 (in Japanese).

上に3項目に亘り種々記載したが(1)は今後斯る記號を一般に及ぼす時常に問題となるから今から都合のよい方を確立して置き度いものであり，(2) は單に小生の考へ易からんと思ふ説明を試みたものであり，(3)に於ては少くとも著者の所謂單位インピーダンスなる觀念より特有インピーダンスなる觀念を導き入れた方が都合が好いのではないかと思惟せられ淺學菲才を顧ず敢て本稿を草した次第である。

同上に對する答辯

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（日本電氣株式會社）

繼電器回路網は實用的方面に於て現在實に見事な發達を遂げて居ますが，夫の解析的研究方法及び構成方法としては遺憾乍ら準據すべき理論もなく單に經驗に依る所謂定石を集積し一つ一つの場合毎に頭腦を酷使して居たと見ても差支ない樣な狀態であります。私共はかゝる狀態に鑑み，繼電器回路網に關する記號式表現と演算に依る一つの理論體系を組立てゝ，回路網の基本的な性質を闡明し解析的研究方法並に構成方法に就て一般的な取扱ひ方を確立し，かくして一定の科學性を與へる事に依つて思考力の經濟化を實現しようと念願してゐるのであります。併し乍ら斯の樣な企ては各方面の御叱正御協力に依つて始めて可能なるべく，從來本誌上に發表して來た拙稿に於ても此の點を强調し御願してゐる次第であります。今御懇篤な御檢討を賜り誠に欣快に堪へません。以下貴討論の順序に從つて御答へ致します。

I. 先づインピーダンス函數の展開式の一つの價値として形式の美しさを强調して居られるやうに承りますが，私共としては一般的な展開式を見出す事に依つて，(1)或る特定の關係を持つ回路網相互間の一般的性質を明かにする事に依つて一般的な洞察に便ならしめ，(2) 展開式の各項に示す如く考慮に取入れるべきものの限界を明示して考へ落しのない樣に取扱者に安心を與へ，(3) 與へられた問題に對しては與へられた條件に從つて一般展開式の各項の係數値を定めると云ふ簡單な謂はば機械的な操作で解を求める事，等を根本目標とし斯くして思考力の經濟化を圖らうとしたのであります。即ち實用的意義を認めて行つたのでありまして，一般展開式が簡易な對稱函數の形で求められた事は此の企圖を助けて與れた結果とはなりますが，決して單なる形式の美を樂しむ數學的趣味から出發したものではありません。此の點はよく御承知下さつて居られる事とは存じますが，一言私共の方針を述べさせて戴きます。

II. Z_α, Z_β 等の定義の仕方に依つて記號式の演算形式が異り，α, β 等の suffix に着目した場合には集合の和及び積の觀念と關聯して式の演算が考へ易い事は御說の通りであります。然らば何故に私共が御說と反對の定義を與へたかと申しますと，回路網構成に際する從來の一般的な思考的慣習に重點を置いたからであります。受動網の目的は作動素の作動狀態を制御するに在り，一般に多くの場合復舊狀態に在る作動素を動作させる點に着目すると云ふ思考上の慣習が行はれてゐる樣に存じます。此の場合は受動網のインピーダンス値としては0値卽ち s に着目する事になります。卽ちかゝる慣習に從へば私共の採つた形式の定義が考へ易いと思ひます。Z_α 等の記號式表現並に演算の目的も窮極する處はかゝるインピーダンスを呈する回路網構成を論ずる點に在り，回路網としても前記の思考的慣習に從つた方が考へ易く，少しの演算上の不便よりも實際的問題として從來の考へ方との思考的摩擦を避けた方がよいのではないか，と考へた譯でありまして，是は各方面の御批判も承りたいと存じてゐる次第であります。

III. 集合の要素としてインピーダンス値の ∞ 値點を採るか0値點を採るかの問題に就きましては，私共としましても最初に可成り考慮した點で次の樣に考へて居ります。數學的方法論としては何れを要素に採つても差支ない事は御說の通りで，記號式表現及び演算の形式としては簡明な双對性の關係に歸し得る性質のものであります。然らば何故に私共が一貫して ∞ 値點を要素に採つてゐるかと申しますと，一般の傳送回路網に於けるインピーダンス表現式の慣習に從つて無用の思考的摩擦を避けんが爲めであります。

卽ち先づ第一に集合の觀念からすれば和及び積の記號は何も＋及び×に限定せられず數學界で用ひられて

Essentially, S. Okada is suggesting to adopt impedance zero (close contact) as the basic element (corresponding to logical one) rather than impedance ∞ (open contact). Okada thinks this will be easier to understand for most people in general. This means a serial circuit uses AND expression and a parallel circuit uses OR expression. Okada also mentions that H. Piesch (see page below) uses this system.

The answer of Nakashima and Hanzawa is that to adopt impedance ∞ (open contact) is natural way and the expressions of impedance of transmission lines are similar to this though either system will do as they are dual from mathematical point of view.

It should be noticed that C. Shannon adopted the same system as Nakashima and Hanzawa did.

(Comment by Prof. Akihiko Yamada)

5 References to the Publications of Nakashima and Hanzawa

This section presents first pages and the pages with references to the work of A. Nakashiima and M. Hanzawa of the following papers

1. Shannon, C.E., "The synthesis of two-terminal switching circuits", Bell System Tech. J., 28, No. 1, 1949, pages 59 and 98 (pages 175 and 176).
2. Piesch, H., "Begriff der allgemeinen Schaltungstechnik", *Archiv für Elektrotechnik*, Berlin, E.T.Z. Verlag, Vol. 33, Heft 10, 1939, 672-686, (in German), page 672 and 673 (pages 177-178).
3. Plechl, O., Duschek, A., "Grundzüge einer Algebra der elektrischen Schaltungen", Ö. Ing.-Archiv, Springer Verlag Wien, 1946, Bd I, H.3, 203-230, pages 203 and 204 (pages 179-180).
4. Church, A., "Review of Nippon Electrical Communication Engineering, by Akira Nakasima and Masao Hanzawa", *The Journal of Symbolic Logic*, Vol. 18, No. 4, 1953, 346 (page 182).
5. Church, A., "Archiv für Electrotechnik - review of papers", *The Journal of Symbolic Logic*, Vol. 30, No. 2, 1952, 247-248 (page 183).
6. Hanzawa, M., "Origin and Development of Switching Circuit Theory", Chapter 2 in *Switching Circuit Theory*, Nippon Electric Company (NEC), July 17, 1989, pages 7, 8, 15 (pages 184-187).

At pages 188 to 193, we show pages 327 and 328 as well as the corresponding references from

Tsutomu Sasao, *Switching Theory for Logic Synthesis*, Kluwer Academic Publishers, 1999, Appendix A, *History of Switching Theory*, Appendix A7, *Switching Theory in Japan*,

where the work of Akira Nakashima and Masao Hanzawa has been discussed in the context of history of Switching theory in Japan.

The Synthesis of Two-Terminal Switching Circuits

By CLAUDE. E. SHANNON

PART I: GENERAL THEORY

1. Introduction

THE theory of switching circuits may be divided into two major divisions, analysis and synthesis. The problem of analysis, determining the manner of operation of a given switching circuit. is comparatively simple. The inverse problem of finding a circuit satisfying certain given operating conditions, and in particular the *best* circuit is, in general, more difficult and more important from the practical standpoint. A basic part of the general synthesis problem is the design of a two-terminal network with given operating characteristics, and we shall consider some aspects of this problem.

Switching circuits can be studied by means of Boolean Algebra.[1,2] This is a branch of mathematics that was first investigated by George Boole in connection with the study of logic, and has since been applied in various other fields, such as an axiomatic formulation of Biology,[3] the study of neural networks in the nervous system,[4] the analysis of insurance policies,[5] probability and set theory, etc.

Perhaps the simplest interpretation of Boolean Algebra and the one closest to the application to switching circuits is in terms of propositions. A letter X, say, in the algebra corresponds to a logical proposition. The sum of two letters $X + Y$ represents the proposition "X or Y" and the product XY represents the proposition "X and Y". The symbol X' is used to represent the negation of proposition X, i.e. the proposition "not X". The constants 1 and 0 represent truth and falsity respectively. Thus $X + Y = 1$ means X or Y is true, while $X + YZ' = 0$ means X or (Y and the contradiction of Z) is false.

The interpretation of Boolean Algebra in terms of switching circuits[6,8,9,10] is very similar. The symbol X in the algebra is interpreted to mean a make (front) contact on a relay or switch. The negation of X, written X', represents a break (back) contact on the relay or switch. The constants 0 and 1 represent closed and open circuits respectively and the combining operations of addition and multiplication correspond to series and parallel connections of the switching elements involved. These conventions are shown in Fig. 1. With this identification it is possible to write an algebraic *and $S_k(X_1, X_2, \cdots, X_m)$ is the symmetric function of $X_1, X_2, \cdots, X_n$ with k for its only a-number.*

This theorem follows from the fact that since f is symmetric in $X_1, X_2, \cdots, X_m$ the value of f depends only on the *number* of X's that are zero and the values of the Y's. If exactly K of the X's are zero the value of f is therefore f_K, but the right-hand side of (6) reduces to f_K in this case, since then $S_j(X_1, X_2, \cdots, X_m) = 1, j \neq K$, and $S_K = 0$.

The expansion (6) is of a form suitable for our design method. We can realize the disjunctive functions $S_K(X_1, X_2, \cdots, X_n)$ with the symmetric function lattice and continue with the general tree network as in Fig. 24, one tree from each level of the symmetric function network. Stopping the trees at Y_{n-1}, it is clear that the entire network is disjunctive and a second application of Theorem 1 allows us to complete the function f with two elements from Y_n. Thus we have

Theorem 16. Any function of $m + n$ variables symmetric in m of them can be realized with not more than the smaller of

$$(m + 1)(\lambda(n) + m) \text{ or } (m + 1)(2^n + m - 2) + 2$$

elements. In particular a function of n variables symmetric in $n - 2$ or more of them can be realized with not more than

$$n^2 - n + 2$$

elements.

If the function is symmetric in $X_1, X_2, \cdots, X_m$, and also in $Y_1, Y_2, \cdots, Y_r$, and not in $Z_1, Z_2, \cdots, Z_n$ it may be realized by the same method, using symmetric function networks in place of trees for the Y variables. It should be expanded first about the X's (assuming $m < r$) then about the Y's and finally the Z's. The Z part will be a set of $(m + 1)(r + 1)$ trees.

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6. C. E. Shannon, "A Symbolic Analysis of Relay and Switching Circuits," *Trans. A. I. E. E.*, V. 57, p. 713, 1938.
7. J. Riordan and C. E. Shannon, "The Number of Two-Terminal Series Parallel Networks," *Journal of Mathematics and Physics*, V. 21, No. 2, p. 83, 1942.
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9. H. Piesch, Papers in Archiv. from *Electrotechnik* XXXIII, p. 692 and p. 733, 1939.
10. G. A. Montgomerie, "Sketch for an Algebra of Relay and Contactor Circuits," *Jour. I. of E. E.*, V. 95, Part III, No. 36, July 1948, p. 303.
11. G. Pólya, "Sur Les Types des Propositions Composées," *Journal of Symbolic Logic*, V. 5, No. 3, p. 98, 1940.

Strecke und der Kondensator entlädt sich über einen stromstarken Lichtbogen vollständig. Eine andere Art der Wiederzündung ist durch Thermoionisierung mehrere Mikrosekunden nach erfolgter Stromunterbrechung möglich. Nachdem der Strom zu Null geworden ist, brennt der Metalldampf weiter und kann durch die wirksam werdende Verbrennungswärme über einer Temperatur von 3500° C erhitzt werden, wobei er durch Thermoionisierung leitend wird; dann vermag die am Kondensator zurückgebliebene Restspannung selbst über verhältnismäßig große Strecken einen stromstarken Lichtbogen einzuleiten, wodurch sich der Kondensator vollständig entlädt. Ob diese Wiederzündung einsetzt oder ob die Restspannung am Kondensator bestehen bleibt, hängt von der Höhe derselben, von der Drahtlänge und insbesondere vom Leitermaterial ab; je höher die Verbrennungswärme eines Metalls, um so leichter erfolgt eine Wiederzündung.

Die vorliegende Arbeit wurde im Institut für Starkstrom- und Hochspannungstechnik der T. H. Dresden durchgeführt. Für die vielen wertvollen Anregungen und Unterstützungen erlaube ich mir, Herrn Prof. Dr.-Ing. Ludwig Binder meinen ergebensten Dank auszusprechen.

Begriff der allgemeinen Schaltungstechnik.

Von

Hansi Piesch, Berlin.

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DK 621.3.06.001.3 + 621.316.3.021/.022

Übersicht. Aus den Aufgaben der elektrischen Schaltungstechnik werden diejenigen Gruppen als „allgemeine Schaltungstechnik" zusammengefaßt, welche sich ausschließlich mit den Verbindungen zwischen den einzelnen Verbrauchern und den Energiespeichern befassen. Für diese Gruppen wird eine Systematik entwickelt, welche den Aufbau, ebenso wie die gedankliche Zergliederung der „allgemeinen Schaltungen" wesentlich erleichtert.

Diese Arbeit ist allgemeinen schaltungstechnischen Fragen gewidmet, die hier von einheitlichen Gesichtspunkten aus behandelt und allgemein gültigen Lösungen zugeführt werden sollen[1]).

Der Aufgabenbereich der Schaltungstechnik berührt nahezu alle Zweige der Elektrotechnik: Beim Bau von Kraftanlagen, ebenso wie von Fernmeldezentralen, Signaleinrichtungen; an Vakuumröhrensendern ebenso wie für elektrisch betriebene Maschinen müssen Schaltungen entwickelt und ausgearbeitet werden. Die Fragen, welche dem Schaltungstechniker bei der Lösung dieser Aufgaben erwachsen, können in zwei grundsätzlich verschiedene Gruppen geteilt werden; diese beziehen sich einerseits auf:

1. die Schaltung der Energiequellen und Verbraucher, anderseits auf:
2. die Herstellung der leitenden Verbindungen zwischen Energiequellen und Verbrauchern.

Während die Schaltung der Stromquellen und Verbraucher in jedem Zweig der Elektrotechnik gesondert behandelt werden muß, ist dies bei der zweiten Gruppe der schaltungstechnischen Aufgaben keineswegs der Fall. Wenn es sich z. B. darum

[1]) Die Arbeit wurde entwickelt aus: 1. Unveröffentlichten Arbeiten von O. Plechl, Wien. 2. Der Abhandlung: „The Theory of equivalent transformation of simple partial paths in the relay circuit". Nakasima, A. u. M. Hanzawa Nippon electr. Comm. Engng. 9 (Febr. 1938) S. 32.

handelt, eine bestimmte Anzahl von Verbrauchern gleichzeitig oder in einer vorgegebenen Reihenfolge anzuspeisen, oder wenn die Anspeisung von verschiedenen Stellen der Anlage unter vorgegebenen Bedingungen ermöglicht, bzw. verhindert werden soll, so müssen zur Lösung dieser Aufgaben dieselben grundsätzlichen Überlegungen angestellt werden; gleichgültig, welche Leistungen umgesetzt und welche Art von Verbrauchern angeschlossen werden sollen.

Sinngemäß sollen nun diese Fragen, welche sich nur auf die Schaltung zwischen Verbrauchern und Stromquellen beziehen, als „Allgemeine Schaltungstechnik" zusammengefaßt werden. Mit dem Wort „Schaltung" soll hier demnach ausschließlich die Gesamtheit der Stromwege bezeichnet werden, welche ein- oder mehrere Stromquellen mit den zugehörigen Verbrauchern verbinden. Als wesentliche Bestandteile der Schaltung sind, abgesehen von den Schaltgegenständen wie Stromquellen und Verbrauchern die Schaltwerke zu nennen, durch die die Stromwege über Verbindungsleitungen wahlweise geschlossen oder unterbrochen werden können. In jeder Schaltung ist mindestens ein derartiger Schalter vorgesehen, mit dem die Anlage in Betrieb gesetzt werden kann.

Diese Schaltwerke haben die Aufgabe, die Stromwege dann und nur dann zu schließen, bzw. zu unterbrechen, wenn bestimmte „Schaltbedingungen" erfüllt sind. Eine derartige Bedingung lautet z. B.: Der Strompfad ist geschlossen entweder, wenn das Schaltwerk a in Stellung i und das Schaltwerk b in Stellung k steht, oder wenn a in Stellung m, b jedoch nicht in Stellung n steht.

Da immer nur eine endliche Anzahl von Schaltern verwendet wird und jeder Schalter nur eine endliche Zahl von Stellungen besitzt, ist auch die Zahl der Bedingungen, welche an derartige Schaltungen gestellt werden können, beschränkt.

Beziehen sich diese Bedingungen nur auf einen Schalter, so können nur zwei Arten von Forderungen gestellt werden:

a) Ein Stromweg ist nur dann geschlossen, wenn der Schalter in einer bestimmten Stellung, z. B. in i steht. b) Ein Stromweg ist nur dann geschlossen, wenn der Schalter in einer bestimmten Stellung, z. B. in k, nicht steht. Im Fall a) wird der Stromweg über den einen Kontakt i des Schalters geführt; im Fall b) muß der Stromweg über alle parallel geschalteten Kontakte von b mit Ausnahme des Kontaktes k geführt werden.

Beziehen sich die beiden Forderungen a) und b) auf die gleiche Schalterstellung, so verhalten sich die Forderungen invers zueinander: d. h.: Es können nie beide Forderungen gleichzeitig erfüllt oder gleichzeitig nicht erfüllt sein; jedoch ist bei jeder möglichen Stellung des Schalters eine der beiden Forderungen a) oder b) sicher erfüllt.

Soll der Stromweg über zwei Schalter geschlossen werden, so sind unter den Bedingungen, welche an diese Schaltung gestellt werden können, vier Gruppen zu unterscheiden:

1. Sowohl für den einen als auch für den anderen Schalter (z. B. a und b) ist eine bestimmte Stellung vorgeschrieben.

2. Es muß entweder von dem einen oder von dem anderen Schalter eine bestimmte Stellung eingenommen werden.

3. Es darf entweder von dem einen oder von dem anderen Schalter eine bestimmte Stellung nicht eingenommen werden; d. h.: Die Stellung von Schalter a oder von b muß sich invers verhalten (oder kurz: invers stehen) zu einer vorgegebenen Stellung.

4. Es darf sowohl Schalter a als auch b eine bestimmte Stellung nicht einnehmen; oder mit anderen Worten: Sowohl a als auch b müssen zu bestimmten Stellungen invers stehen.

Soll ein Stromweg dann und nur dann geschlossen sein, wenn die Forderung 1 erfüllt ist, müssen die beiden Kontakte, welche bei den vorgeschriebenen Schalter-

von Schönbach bei Eger. Neues Jb. f. Mineralogie 84, Abt. B, 90—116 (1940). — Einiges über die Untersuchung der geoelektrischen Blitzgefährdung. Gerlands Beitr. Geophys. 57, 65—108 (1940). — Die Möglichkeit einer geoelektrischen Fehlmutung, mit besonderer Berücksichtigung der bei der Funkmutung gegebenen Voraussetzungen. Elektr. im Bergb. 16, Nr. 3, 37—43 (1941). — La smorzamento delle onde Hertziane attraverso i conduttori geologici. Geofisica pura e applicata 4, Nr. 1, 15—37 (1942). — und Heinrich Forejt: Nachweis steil einfallender elektrischer Diskontinuitätsflächen im Untergrunde durch Funkmutung. Hochfrequenztechn. u. E.-A. 59, Nr. 2, 41—45 (1942). — und Heinrich Forejt: Die Anwendung des Druckindikators in der Funkmutung. Z. Geophysik 17, Nr. 5/6, 217—225 (1941/42). — Die Aussichten der Reflexionsmethode in der Funkmutung. Glückauf 79, Nr. 27/28, 336—340 (1943). — Wassersuche durch Funkmutung. Wasserkr. u. Wasserwirtsch. 38, Nr. 9, 209—215 (1943). — Mitteilung über die im Blitzversuchsfeld Absroth im Jahre 1941 durchgeführten Arbeiten. Gerlands Beitr. Geophys. 59, Nr. 3/4, 306—330 (1943). — Nachweis von Schwimmsandvorkommen durch geoelektrische Verfahren. I. Teil. Braunkohle 44, Nr. 1/2, 6—9 (1945). — Einiges über den Blitzschutz besonders gefährdeter Objekte. Z. ges. Schieß- u. Sprengstoffwes. 39, Nr. 7, 8, 9, 101—108 (1944). — Meßverfahren der Funkmutung. München: Verlag Oldenbourg. 1943.

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Grundzüge einer Algebra der elektrischen Schaltungen.

Von O. Plechl und A. Duschek, Wien.

Mit 29 Textabbildungen.

Übersicht.

Die vorliegende Arbeit zeigt die Möglichkeit einer rechnerischen Behandlung der elektrischen Schaltungen, die allerdings erst die Entwicklung eines im wesentlichen völlig neuen algebraischen Kalküls erfordert. Nach einer Definition der Grundbegriffe in 1. werden die Grundlagen des Kalküls dargestellt und an Beispielen erläutert. Zum Schluß (11.) wird noch auf den Zusammenhang mit dem Aussagenkalkül der algebraischen Logik eingegangen.

Elektrische Schaltungen sind die Verbindungsglieder zwischen Stromquellen und Verbrauchern, worunter wir alle elektrischen Maschinen und Apparate verstehen, in welchen eine Energieumwandlung stattfindet. Die Aufgabe der Schaltung ist es, gewisse Gruppen von Verbrauchern an gewisse Gruppen von Stromquellen anzuschließen oder sie von diesen abzuschalten, d. h. also, gewisse Strompfade herzustellen oder zu unterbrechen. Die Strompfade müssen vorgeschrieben sein, mitunter ist auch vorgeschrieben, daß die Herstellung oder Lösung der Strompfade von bestimmten verschiedenen Stellen aus zu erfolgen hat. Ein einfaches Beispiel hierfür ist die Wechselschaltung, bei der ein Verbraucher (Glühlampe) von zwei oder mehr verschiedenen Stellen aus ein- oder ausgeschaltet werden kann. Aus diesen Schaltbedingungen, die den gegebenen Ausgangspunkt bilden, hat dann der Konstrukteur die Schaltung selbst, d. h. ihre graphische Darstellung, das Schaltbild zu ermitteln. Der Weg dazu bestand bisher ausschließlich im Probieren, wobei natürlich dem Konstrukteur seine mehr oder minder große praktische Erfahrung, seine Routine, zu Hilfe kommt. Irgendeine Systematik, ja auch nur der Ansatz zu einer solchen fehlte bisher vollständig, wenn man von den Ansätzen von Edler[1] und Lischke[2] absieht, und daher war man

[1] R. Edler: Der Entwurf von Schaltungen und Schaltapparaten (Schaltungstheorie), 2 Bände. Leipzig. 1905 und 1927.

[2] Lischke: Schaltlehre. Leipzig. 1921.

auch nicht in der Lage zu erkennen, ob eine einmal gefundene Lösung auch die einfachste und zweckmäßigste sei. Meist ist man ja froh, überhaupt eine Lösung der manchmal recht komplizierten Schaltbedingungen zu haben.

Der Gedanke einer mathematischen Behandlung der elektrischen Schaltungen mag fürs erste recht befremdend wirken, da die Möglichkeit eines allgemeinen Ansatzes nicht ohne weiteres erkennbar ist. Nun gibt es zwar ein Gebiet der Mathematik, das sich in eine sehr enge Beziehung zu den Schaltungen bringen läßt, und zwar ist das der Aussagenkalkül der algebraischen Logik. Aber einerseits ist dieser Kalkül selbst vielen Mathematikern und erst recht den Praktikern kaum vom Hörensagen bekannt, anderseits geht das Interesse des Logistikers in einer ganz anderen Richtung als das unsere, so daß der Aussagenkalkül, dessen ganzer Ausbau derzeit überhaupt noch etwas dürftig erscheint, nur wenig mehr als eben die bloße Erkenntnis bringt, daß eine mathematische Theorie der elektrischen Schaltungen überhaupt möglich ist und daß es sich dabei um eine Art Algebra handelt. Wir werden im folgenden diese Algebra der elektrischen Schaltungen unabhängig vom Aussagenkalkül entwickeln und auf den Zusammenhang mit diesem erst zum Schluß kurz eingehen.

Die Hauptschwierigkeit jeder Anwendung der Mathematik liegt im Übergang von dem konkreten, in Worten formulierten physikalischen oder technischen Problem zu seiner mathematischen Fassung, also im mathematischen Ansatz. Dasselbe gilt natürlich auch von der Schaltalgebra, obwohl hier bei gewissen Schaltbedingungen, wenn die Zahl der Schalter vorgeschrieben ist, der mathematische Ansatz ungemein einfach ist. Auf der anderen Seite aber sind dem Schaltungstechniker die einfacheren, mitunter aber auch kompliziertere Schaltungen so geläufig, daß ihm in solchen Fällen die mathematische Behandlung keine Vorteile mehr geben kann. Wir glauben aber doch, daß es verfehlt wäre, die ganze Schaltalgebra von vornherein aus diesem Grunde als überflüssig abzulehnen. Abgesehen davon, daß schon die Möglichkeit einer solchen Disziplin an sich von größtem prinzipiellen Interesse sein dürfte, besteht gar kein Zweifel, daß der Kalkül, wenn man sich einmal mit ihm richtig vertraut gemacht hat, eine wertvolle Bereicherung des mathematischen Rüstzeuges des Elektrotechnikers darstellt und zu durchaus neuen, praktisch wichtigen Gesichtspunkten führt.

Die leitende Idee einer mathematischen Behandlung der Schaltungen, die Erkenntnis ihrer Möglichkeit und die Entwicklung des Kalküls, zunächst für eigene, rein praktische Zwecke stammt ausschließlich von Dr. O. Plechl. Er hat bemerkt, daß man sehr schön mit den Kontakten rechnen kann, wenn man sie mit irgendwelchen Buchstaben $a, b, \ldots$ bezeichnet und dann die Parallelschaltung durch $a + b$, die Serienschaltung durch $a \cdot b$ wiedergibt. Etwas ähnliches machen auch Nakasima und Hanzawa,[3] die aber die Parallelschaltung mit $a \cdot b$ und die Serienschaltung mit $a + b$ bezeichnen. Das ist natürlich ganz unwesentlich und bedeutet von unserem Standpunkt aus nur, daß an Stelle der Leitwerte die Widerstandswerte der Rechnung zugrunde gelegt werden. Plechl hat weiter die formalen Gesetze der auf diesen beiden Grundoperationen aufgebauten Algebra gefunden und den Kalkül auf die verschiedensten Probleme der Schaltungstechnik angewendet. Die Begriffe Grenzfunktion, Vollform und Kürzung, all das stammt von Plechl, über dessen Gedankengang und Rechenmethoden in mancher Hinsicht die Arbeiten von H. Piesch[4, 5] besseren Aufschluß geben als die vorliegende, in der der Bearbeiter vor allem durch die grundsätzliche Einführung des Leitwertes einer Admittanz dem ganzen Kalkül eine ein-

[3] A. Nakasima und M. Hanzawa: The theory of equivalent transformation of simple partial paths in the relay circuit. Nippon electr. Commun. Engng. **9**, 32 (1938).

[4] H. Piesch: Begriff der allgemeinen Schaltungstechnik. Arch. Elektrotechn. **33**, 672 (1937).

[5] H. Piesch: Über die Vereinfachung von allgemeinen Schaltungen. Arch. Elektrotechn. **33**, 672 (1937).

Nippon Electrical Communication Engineering. by Akira Nakasima; Masao Hanzawa
Alonzo Church
The Journal of Symbolic Logic, Vol. 18, No. 4. (Dec., 1953), p. 346.

Archiv fur Elektrotechnik by Hansi Piesch
Alonzo Church
The Journal of Symbolic Logic, Vol. 30, No. 2. (Jun., 1965), pp. 247-248.

Alonso Church was a founder of the *Journal of Symbolic Logic* in 1936 and was an editor of the reviews section from beginning until 1979. In the fourth volume of the Journal Chutrch published a paper *A Bibliography of Symbolic Logic*, and viewed the reviews section as a continuous of this work.

Alonso Church had very important contirbutions in mathematical logic, recursion theory, and theoretical computer science, well known as a founder of *the Lambda calculus*, and the *Church theorem* and *Church thesis*, both of which appeared in the first volume of the *Jounral of Symbolic Logic*.

ship was made in 1910 by Erénfést, in his review ($186\frac{1}{2}$**1**) of Couturat's 100**20**A. According to Ánovskaá (XVI 46), details of Erénfést's proposal were worked out by Šéstakov in 1934–35 but not published until 1941. Meanwhile the same idea had been reached independently by Nakasima and Hanzawa in 1936 (see the review next following).

The reviewer has not seen Erénfést's review, and is indebted to George L. Kline for information as to its content. He is also indebted to George W. Patterson for calling his attention to the papers of Nakasima and Hanzawa, and others published in Japan.

ALONZO CHURCH

AKIRA NAKASIMA and MASAO HANZAWA. *The theory of equivalent transformation of simple partial paths in the relay circuit.* ***Nippon electrical communication engineering*** (Tokyo), no. 9 (February 1938), pp. 32–39.

AKIRA NAKASIMA. *The theory of four-terminal passive networks in relay circuit.* Ibid., no. 10 (April 1938), pp. 178–179.

AKIRA NAKASIMA. *Algebraic expressions relative to simple partial paths in the relay circuit.* Ibid., no. 12 (September 1938), pp. 310–314.

AKIRA NAKASIMA. *The theory of two-point impedance of passive networks in the relay circuit.* Ibid., no. 13 (November 1938), pp. 405–412.

AKIRA NAKASIMA. *The transfer impedance of four-terminal passive networks in the relay circuit.* Ibid., no. 14 (December 1938), pp. 459–466.

AKIRA NAKASIMA and MASAO HANZAWA. *Expansion theorem and design of two-terminal relay networks (Part I).* Ibid., no. 24 (April 1941), pp. 203–210.

Nippon electrical communication engineering publishes condensed English translations, and abstracts in English, of papers which were previously published in Japanese in the ***Journal of the Institute of Electrical Communication Engineers of Japan.*** The first of the above papers, for example, is described as a condensed translation of a paper which appeared in two parts in the latter periodical, no. 165 (December 1936) and no. 167 (February 1937). The reviewer has not seen the Japanese originals of the papers.

The six papers are concerned with developing and applying an algebra of partial paths in relay circuits, which is in fact identical with the "symbolic relay analysis" that was later introduced by Shannon, and dual to the "algebra of switching circuits" of Erénfést and Šéstakov (see the preceding review).

The first paper introduces the algebra by providing that if A and B are simple partial paths (two-terminal circuits), then $A + B$ shall represent the series connection of A and B, and AB the parallel connection of A and B; $A = B$ shall mean that the acting functions of A and B are equal, i.e., that A is open when B is open and closed when B is closed; $\bar{A}$ shall be a simple partial path which is open when A is closed and closed when A is open; p and s shall be simple partial paths which are always open and always closed respectively (or, as the authors say, give always infinite impedance and zero impedance respectively). Many laws of the algebra are developed which in fact coincide with familiar laws of Boolean algebra, but the authors do not state that the algebra is a Boolean algebra.

In the third paper (of which the Japanese version was published in August 1937) the algebra is reduced to an algebra of sets by making correspond to each simple partial path the set of (in effect) times at which its impedance is infinite, so that "theorems and expressions developed in the theory of set may, therefore, be applied to acting impedance problems of simple partial paths." In the sixth paper the authors make explicit reference for the first time to Boole (19**3**) and Schröder (42**7**); the expansion theorem mentioned in the title of this paper is Boole's law of development (19**3**, pp. 72–75), as the authors point out.

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Archiv fur Elektrotechnik

Review Author[s]:
Alonzo Church

The Journal of Symbolic Logic, Vol. 30, No. 2 (Jun., 1965), 247-248.

HANSI PIESCH. *Begriff der allgemeinen Schaltungstechnik.* **Archiv für Elektrotechnik,** vol. 33 (1939), pp. 672–686.

HANSI PIESCH. *Über die Vereinfachung von allgemeinen Schaltungen.* Ibid., pp. 733–746.

These early papers about switching algebra are of very considerable historical interest. As his sources the author refers to unpublished work of O. Plechl (Vienna) and to Nakasima and Hanzawa XVIII 346(1). Like Nakasima and Hanzawa, he fails to observe that the algebra is in fact Boolean (or better, propositional) algebra. And various ideas of the nineteenth-century algebra of logic are worked out afresh, and evidently independently.

Not only two-position switches but switches having any finite number of positions are dealt with, and not necessarily all of them the same number of positions. Letters such as $a, b, c, \ldots$ are used for switches, and different positions of a switch are indicated by subscripts. In effect a_0 is used to express the proposition that the switch a is in its first position (the position in which its first contact is closed), and b_3 that the switch b is in its fourth position, and so on. Capital letters may be used to express other propositions, e.g. that a certain light or a certain motor is turned on. And the expressions of the algebra are built up out of these propositional letters by means of the signs for multiplication (i.e., conjunction), addition (i.e., disjunction), and the inverse (i.e., negation). The conditions given in any particular problem will be expressed as conditions on the propositional letters in this notation; and besides the special conditions of a particular problem we must always take into account also all

Continuation at page 248

conditions of the form

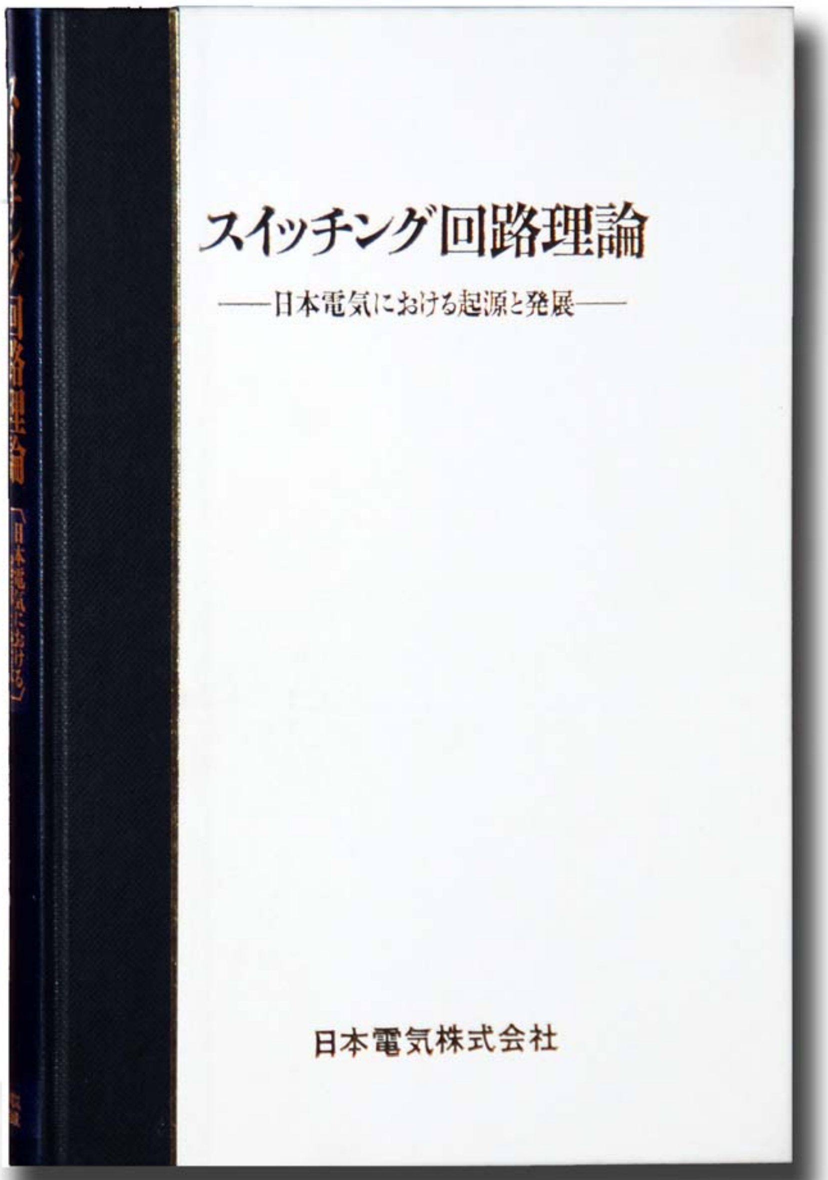
$$\bar{a}_i = a_0 + a_1 + \ldots + a_{i-1} + a_{i+1} + \ldots + a_{n-1},$$

where a is any n-position switch.

The author's system is propositional algebra rather than propositional calculus, since only equivalences are asserted. It is, moreover, ordinary two-valued propositional algebra, since every expression of the algebra has (in effect) one or other of two truth-values, truth or falsehood. And it is indeed the reviewer's impression that the author's method of dealing with multiple-position switches can be expected to work more smoothly than proposals which have sometimes been made to use many-valued propositional calculus for this.

Errata: in line 14 from the bottom of page 674, it seems probable that a_i was meant to have a bar over it; at several places on page 675, a_l is evidently a misprint for a_1, and ce a misprint for c_e. ALONZO CHURCH

Switching Circuit Theory, NEC, July 17, 1989.



Masao Hanzawa, "Origin and Development of
Switching Circuit Theory", Chapter 2 in *Switching Circuit Theory*,
Nippon Electric Company (NEC), July 17, 1989.

2　スイッチング回路理論の起源と発展

2.1　スイッチング回路理論の起源

中嶋章らによって，今から半世紀前の昭和10(1935)年頃，世界にさきがけて，現在のディジタル技術の基本であるスイッチング回路理論の研究が始められた[1]。

この当時，通信では伝送技術の研究と実用化が盛んな頃であり，それも振幅変調による多重通信技術の花が開いた時であった。そして，その理論的な処理には一般交流回路理論が，複素変数函数論などを基礎として定着していた。

一方，交換技術は当時ステップバイステップ方式が中心であり，その理論的な背景としては回線数・機器数などを決定するためのトラヒック理論が確率論を基礎としてあっただけで，回路設計そのものをバックアップするような理論体系はなかった。

そのため，交換回路技術者は，回路を設計する場合，継電器の動作を順次追って，試行錯誤によって組合せ回路を構築し，また回路動作を読んで解析するなど，多分に記憶と過去の事例などによる経験にたよらざるを得なかった。すなわち，回路図の中の個々の継電器が，動作復旧を繰返す状態をいちいち記憶の中に留めながら，それらを頭の中でつなぎ合せて，全体の回路機能を読んで行かねばならなかった。この極めて非能率でもあり不確かでもある作業は，しばしば囲碁や将棋の“読み”にも喩えられたように，彼らに少

なからぬもどかしさを感じさせていた。

そのために，このような方法から脱し，少しでも合理化をはかりたいという願いが，このスイッチング回路の理論化を彼らにすすめさせる動機となったのは当然であった。

昭和10年の初頭，東京新橋の電気クラブの講堂における，電信電話学会主催の技術講演会で，当時27歳であった日本電気の若い技術者，中嶋の「継電器回路の構成理論」と題する講演が盛会裡に延々３時間にわたって行われた。この講演の内容は，中嶋が当時日本電気において手掛けた自動交換機，遠方監視制御装置などの各種継電器回路装置の設計作業を通じて，それから抽出して得た継電器回路設計の定石の分析結果を理論的にまとめたものが主体であった。そしてそれらの内容の中には，ディジタル技術の基礎であるところの，スイッチング回路理論の基本的な諸定理をも含んでいたのである。中嶋自身はこの時点ではもちろん，後年の輝かしいディジタル化時代が訪れるであろうことも，またその時代への幕開けを自らが演じていることも知ってはいなかったわけである。

中嶋は昭和５年４月，東京帝国大学の電気工学科を卒業後直ちに日本電気に入社し，当時の技師長丹羽保次郎博士の指示で，研究課研究係の中で嶋津保次郎が担当する特殊リレー回路装置研究班に配属された。それから昭和11年に技術課伝送係へ移るまで，嶋津の指導のもとで自動交換機，その他の継電器の応用装置の設計に従事した。これらの設計には新しい継電器回路を作らねばならないものが多数あり，特に学校を出て会社へ入った早々の中嶋には必要以上にそれらが面倒と感じられたのであった。これが中嶋を駆って継電器回路に筋を通そうという考えを起こさせる動機となったのである。

中嶋の研究は初めは主として継電器回路の過渡現象の究明に向けられ，その成果は当時の日電月報に，昭和９年11月から〝継電器工学の理論と実際〟と題して連載された。この論文が注目をあび，前述の講演会が行われ，その予稿が昭和10年９月の同学会誌に発表されたのである。この講演予稿「継電器回路の構成理論」[2] には，現在のスイッチング回路の論理代数的演算法の基本が，定義および定理として述べられ，それらの中には現在ド・モルガンの定理として知られているものまで含まれていたのである。ただ，それらは

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3) 中嶋：「継電器回路における単部分路群の性質について」，信話誌，昭11年2月
4) 中嶋，榛沢：「継電器回路における単部分路の等価変換の理論(其の一)」，信話誌，昭11年12月
5) 中嶋，榛沢：「同上(其の二)」，信話誌，昭12年2月
6) 中嶋：「継電器回路における負性四端子回路網の理論について」，信学誌，昭12年4月
7) 中嶋：「継電器回路における単部分路の関係式論」，電通誌，昭12年8月
8) C. E. Shannon："A Symbolic Analysis of Relay and Switching Circuits," Trans. AIEE 57, p. 713, 1938
9) H. Piesh："Begriff der Algemeinen Schaltungstechnik," Arch. f. Elektrotechnik, Okt. 1939
10) 中嶋：「継電器回路に於ける負性回路網の2点間作動インピーダンスの理論(其の一)」，電通誌，177号，昭12年12月
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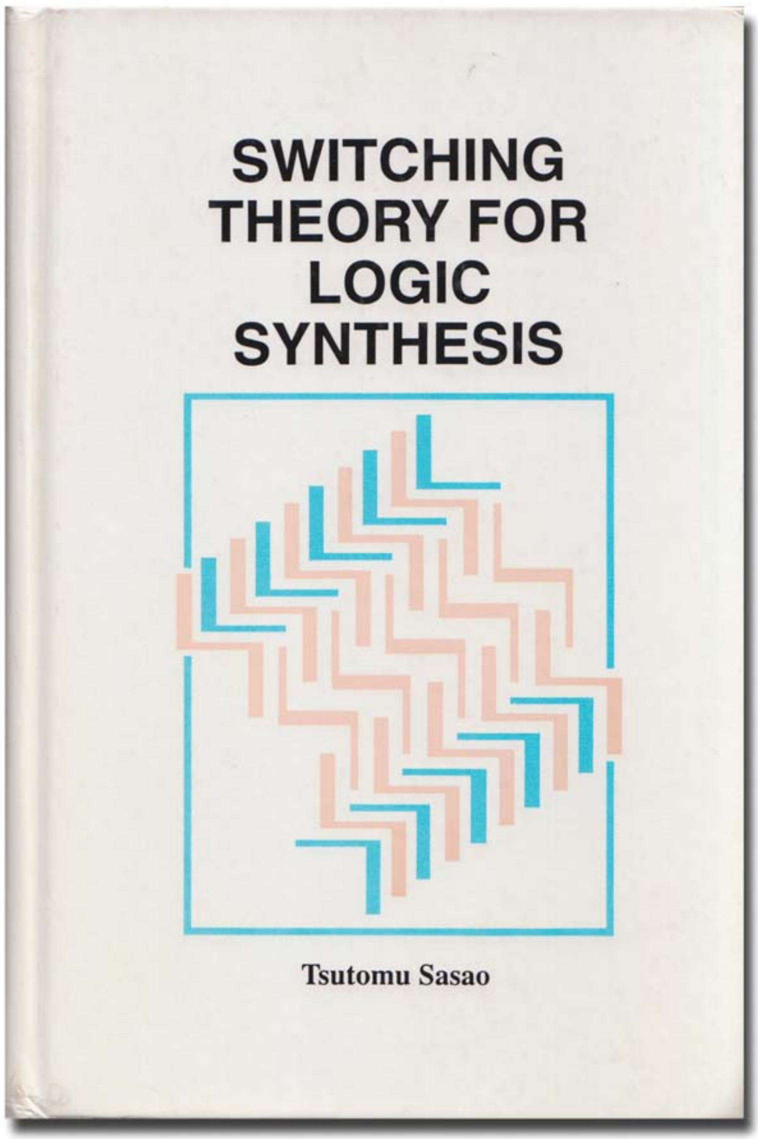
Kluwer Academic Publishers, 1999,
Appendix A, *History of Switching Theory*,
Appendix A7, *Switching Theory in Japan*, page 327



porated this technique. SIS developed at U. C. Berkeley is a tool for sequential and combinational logic synthesis. It supports all the features of MIS.

A.7 SWITCHING THEORY IN JAPAN

In 1930, Akira Nakashima, just graduated Tokyo University, entered NEC (Nippon Electric Company). First, he was assigned to the research group for designing special relay network systems. In this group, he designed automatic exchange systems as well as other relay application systems for six years. During that period, he became interested in the transient analysis of relay networks and formulated the design theory of relay networks. Since November 1934, he published his idea in a series of papers "Theory and practice of relay engineering," in the monthly technical journals of NEC. His papers attracted attention of many people, and the Telegraph and Telephone Society of Japan invited him as a guest speaker at its technical meeting in early 1935. His three-hour talk included design theories of relay networks that he formulated through designing various automatic exchange systems. The pre-print of the talk was published as "Synthesis theory of relay networks," [284] in the *Journal of the Telegraph and Telephone Society of Japan* in September 1935. The paper presented basic switching theory in terms of definitions and theorems, including De Morgan's Theorem. However, they were not represented by symbols. The formulation of these theorems using symbols were completed by A. Nakashima and M. Hanzawa in 1936 and 1937. At that time, they were not aware of Boolean algebra. Later, in August 1938, Nakashima found that his algebra was exactly the same as Boolean algebra.

In 1956, Toshio Ikeda at Fujitsu completed FACOM128, the first commercial relay computer in Japan [116].

Eiichi Goto [143] invented the *parametron* in 1954 when he was a graduate student of Tokyo University. A parametron consists of two magnetic cores and one capacitor, and realizes a three-input majority function [144]. In March 1957, NTT (Nippon Telegraph and Telephone Corp.) developed the Musashino-1, the first parametron computer using 519 vacuum tubes and 5,400 parametrons. Since it was an ILLIAC compatible machine, the computer could run the software of the ILLIAC. Parametron computers were manufactured by Hitachi, Fujitsu and NEC. Especially, NEC sold a few hundred sets [116]. Although parametron computers were reliable, they were very slow: They worked only at the speed of 200 kHz. Thus, after the appearance of reliable transistors, parametron computers became obsolete. Although the parametron computers were sold for only for two years, they made a strong influence in the switching theory. The realization of logic functions using majority elements were the first problem [258, 408]. After that the majority elements were generalized to threshold elements [272, 273], and they attracted many Japanese researchers [11, 145, 430].

Mochinori Goto [142] at the Electro-Technical Laboratory (ETL) analyzed operations of contact relay networks by introducing three-valued logic.

In July 1956, ETL developed a transistor computer, the ETL Mark III, that used 130 transistors and 1,700 diodes. This was the first stored program computer using transistors. They used point-contact transistors whose reliability was quite low. In spite of this difficulty, they completed the computer in two years.

Y. Komamiya [213] in ETL derived relation between arithmetic expressions and Boolean functions. He also considered the Reed-Muller transforms of logic functions. (See Problem A.2 and A.3). I. Ninomiya [298] considered invariant properties of Fourier transformations, and applied them to the classification of switching functions. His main results were included in the Harrison's book [157]. K. Kobayashi [209] considered the notion of almost complete set of logic elements, the details are shown in Mukhopadhyay's book [268].

Switching theory was active in the following universities: Tohoku (S. Noguchi, A. Maruoka [241], T. Higuchi [167], M. Kameyama [193]), Meiji (M. Mukaidono [264]), Tokyo Institute of Technology (Y. Thoma [408]), Nagoya (A. Fukumura), Kyoto (S. Yajima [430], H. Mine [257]), Osaka (H. Ozaki [306], T. Kasami [287], K. Kinoshita [206], T. Kitahashi [207], H. Fujiwara [131], T. Sasao), Hiroshima (N. Yoshida [438]), Kyushu (M. Ito [189]). Among them, S. Yajima's group at Kyoto University produced many researchers: Y. Kambayashi [194], T. Ibaraki [181], H. Hiraishi, K. Inagaki [431], H. Yasuura [436], N. Takagi [398], N. Ishiura [186], S. Minato [254, 255], K. Hamaguchi [155], H. Ochi [303], and others.

In addition to the universities, switching theory was also active in Electrical Technical Laboratories (M. Goto [142], Y. Komamiya [213]), NTT (Z. Kiyasu, S. Muroga [272], K. Ibuki [182]), NEC (A. Nakashima [284], T. Watanabe, S. Naito, T. Nanya [290, 291], T. Yamada [432]). LORES [117, 289] developed by MITSUBISHI converted TTL logic networks into ECL logic networks by local transformation method. PARTHENON [74] developed at NTT translated SFL (Structural Functional description Language) into microprocessors.

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論理數學方程式の繼電器回路網理論への應用 *

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1. 概 說

論理數學とは，論理學上の命題，條件等の論理的關係を式で表わしてその變換を行う數學で，その代數的部門は遠く G. W. Leibniz に發芽を見ると言われて居り，Boole 代數とか命題計算とか呼ばれている**論理代數**が，G. Boole により作られ(1847 年) E. Schröder, B. A. W. Russel, D. Hilbert 氏等を經て發展され現代に至つている。現代では論理代數は數學基礎論，**多値論理學**（眞僞の他に不確定又は確率の概念を含む論理學）等の内に活躍し，記號論理學とも言われている。工學上でも繼電器接點回路の理論に中島章[1]，榛澤正男[2]，島津保次郎[3]等の諸氏によつて應用された。大橋幹一氏[4]は繼電器回路の理論としては繼電器卷線の電流と接點の動作との關係，例えばその時間的遲れを表示し得なければ實際的でないと言う點に着眼し，論理代數とは別箇の一種の演算子法を創始され，更に小島哲氏[5]によつて敷衍された。

本論文に於ては，論理代數方程式の應用の他に，接點の動作の遲れを考慮に入れた**論理函數方程式**を作つてその解法を求め，それによつて與えられた繼電器回路網の動作を時刻 t の函數として求め，又は逆に與えられた動作特性を有する繼電器回路網を構成する方法を示しているのであつて，論理代數に對して大橋博士の指摘された缺點を論理數學の形において補おうとしたものである。これについては既に多少報告[6]したが，その後解法その他に大改良を加えて實用的に直し，更に三値論理學を加味して二値論理の短所を補つた。なお，記號論理學の文獻を贈與された伊藤誠博士に深厚なる謝意を表する。

2. 二値論理數學の基礎的公式

$A, B, \cdots\cdots Z$ なる文字は論理學としては命題や條件を表わし，それの眞なるか僞なるかに從つて 1 とか 0 とかの値を採ると定め，これを命題の**眞理値**と言う。眞理値が 2 種であるからこの論理學を**二値論理學**と稱し，普通行われているのはこれである。繼電器回路の接點の動作を表わす場合には，1 **て閉路**，0 **て開路を指示する**。或繼電器卷線 F の電流が流れる時も 1，流れない時も 0 で表わし，F を時刻 t の函數（0 か 1 かの値を採る）とし，その接點 f の動作が時間 τ だけ遲れるとすると，それを F_τ で表わす。即ち

$$F_\tau \equiv F(t-\tau) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2\cdot1)!$$

特に多くの接點の遲れが等しいか又は區別する必要が無い場合は F_τ の代りに f と書くと便利である。次に F の否定を $\sim F$ で表わし次の如く定義する。

$$\sim F \equiv 1-F \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2\cdot2)!$$

即ち回路の開閉は F と $\sim F$ とは互に反對である。

次に，命題 A と B との論理積を $A \cdot B$ 又は AB で表わし「A と B と」の意味である。A と B とが兩方共に眞（即ち 1）なる場合にのみ AB が眞なりと定める。

今後便宜上 $A, B\cdots\cdots$ **て命題** $A, B\cdots\cdots$ **の眞理値を表わし，眞理値が識別し得るもののみ取扱うとすれば**

$$AB \equiv \min(A, B) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2\cdot3)!$$

で，接點 A, B が直列に成つているものの動作が表わされる。

次に，A と B との**論理和**を $A \vee B$ で表わし「A か B か」の意味である。少くとも A か B か，いずれかが眞なる場合に $A \vee B$ が眞なりと定める。即ち

$$A \vee B \equiv \max(A, B) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2\cdot4)!$$

で，接點 A, B が並列に成つているものの動作が表わされる。

次に「命題 A が眞なる場合には必ず B が眞である」ことを表わすのに $A \rightarrow B$ と記し，**含意**と言う。即ち

$$A \rightarrow B \equiv (\sim A) \vee B \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2\cdot5)!$$

* Application of Logical Mathematics to the Theory of Relay Networks. By Motinori GOTO, Member (Electrotechnical Laboratory). 本稿は昭和 23 年 11 月電氣三學會連合大會講演中より特に寄稿を依頼したものである。

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